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**UNIVERZITET CRNE GORE**

**- Odbor za doktorske studije -**

**- Senatu -**

**O V D J E**

U prilogu dostavljamo Odluku Vijeća Elektrotehničkog fakulteta sa sjednice od 18.04.2024. godine i obrazac **D2**, sa pratećom dokumentacijom, za kandidata MSc **Luku Martinovića**, na dalji postupak.

**Vršilac funkcije DEKANa,**

**Prof. dr Budimir Lutovac**

# ISPUNJENOST USLOVA DOKTORANDA

| OPŠTI PODACI O DOKTORANDU | | | |
|---|---|---|---|
| Titula, ime, ime roditelja, prezime | MSc Luka Jovo Martinović | | |
| Fakultet | Elektrotehnički fakultet Podgorica | | |
| Studijski program | Doktorske studije Elektrotehnike | | |
| Broj indeksa | 1/21 | | |
| **NAZIV DOKTORSKE DISERTACIJE** | | | |
| Na službenom jeziku | Kooperativno upravljanje heterogenim multiagentnim sistemima bez razmjene stanja kontrolera | | |
| Na engleskom jeziku | Cooperative control of heterogeneous multiagent systems without exchange of controller states | | |
| Naučna oblast | Automatika | | |
| **MENTOR/MENTORI** | | | |
| Prvi mentor | Prof. dr Božo Krstajić | Elektrotehnički fakultet, Univerzitet Crne Gore, Podgorica, Crna Gora | Automatika |
| **KOMISIJA ZA PREGLED I OCJENU DOKTORSKE DISERTACIJE** | | | |
| Prof. dr Željko Đurović | | Elektrotehnički fakultet, Univerzitet u Beogradu, Beograd, Srbija | Automatika |
| Prof. dr Božo Krstajić | | Elektrotehnički fakultet, Univerzitet Crne Gore, Podgorica, Crna Gora | Automatika |
| Prof. dr Žarko Zečević | | Elektrotehnički fakultet, Univerzitet Crne Gore, Podgorica, Crna Gora | Automatika |

| **Datum značajni za ocjenu doktorske disertacije** | |
|---|---|
| Sjednica Senata na kojoj je data saglasnost na ocjenu teme i kandidata | 16.09.2022. |
| Dostavljanja doktorske disertacije organizacionoj jedinici i saglasanost mentora | 18.03.2024. |
| Sjednica Vijeća organizacione jedinice na kojoj je dat prijedlog za imenovanje komisija za pregled i ocjenu doktorske disertacije | 18.04.2024. |

| ISPUNJENOST USLOVA DOKTORANDA |
|---|
| U skladu sa članom 38 pravila doktorskih studija kandidat **je** cjelokupna ili dio sopstvenih istraživanja vezanih za doktorsku disertaciju publikovao u časopisu sa **(SCI/SCIE)/(SSCI/A&HCI)** liste kao prvi autor. |
| **Spisak radova doktoranda iz oblasti doktorskih studija koje je publikovao u časopisima sa (upisati odgovarajuću listu)** |

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## RADOVI PUBLIKOVANI U ČASOPISIMA SA SCI LISTE:

1) Luka Martinović, Žarko Zečević, Božo Krstajić, Output containment control in heterogeneous multi-agent systems without exchange of controller states, European Journal of Control, Volume 75, 2024, 100889.
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https://www.sciencedirect.com/journal/european-journal-of-control

2) L. Martinović, Ž. Zečević and B. Krstajić, "Distributed Observer Approach to Cooperative Output Regulation of Multi-Agent Systems Without Exchange of Controller States," in IEEE Access, vol. 11, pp. 81419-81433, 2023.
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3) Luka Martinović, Žarko Zečević, Božo Krstajić, Cooperative tracking control of single-integrator multi-agent systems with multiple leaders, European Journal of Control, Volume 63, 2022, Pages 232-239.
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## RADOVI IZLOŽENI NA KONFERENCIJAMA:

1) L. Martinović, Ž. Zečević and B. Krstajić, "Regulated Output Synchronization of Multi-Agent Systems via Output Feedback," 2022 26th International Conference on Information Technology (IT), Zabljak, Montenegro, 2022, pp. 1-4, doi: 10.1109/IT54280.2022.9743524.

2) L. Martinović, Z. Zečević and B. Krstajić, "Cooperative Tracking Control of Multi-Agent Systems with General Linear Dynamics," 2021 25th International Conference on Information Technology (IT), Zabljak, Montenegro, 2021, pp. 1-4, doi: 10.1109/IT51528.2021.9390134.

3) L. Martinović, Ž. Zečević and B. Krstajić, "Cooperative Output Regulation of Multi-Agent Systems with Single-Integrator Dynamics," 2020 28th Telecommunications Forum (TELFOR), Belgrade, Serbia, 2020, pp. 1-4, doi: 10.1109/TELFOR51502.2020.9306565.

4) L. Martinović, Ž. Zečević and B. Krstajić, "Distributed Control Strategy for Multi-Agent Systems Using Consensus Among Followers," 2020 24th International Conference on Information Technology (IT), Zabljak, Montenegro, 2020, pp. 1-4, doi: 10.1109/IT48810.2020.9070617.

**Obrazloženje mentora o korišćenju doktorske disertacije u publikovanim radovima**

Doktorand MSc Luka Martinović je svoja istraživanja na kojima je zasnovana doktorska disertacija objavio u vidu tri rada, koji su publikovani u renomiranim vodećim međunarodnim časopisima sa SCI/SCIE liste (**Q1**), čiji je kumulativni IMPACT Factor (IF) **10.7**. Kandidat je na sva tri rada **prvi autor**. Dio istraživanja je publikovan i u radovima (ukupno četiri) koji su izloženi na međunarodnim IEEE konferencijama. Potrebno je istaći i da je u toku izrade doktorske disertacije doktorand publikovao i druge radove koji su izvan oblasti disertacije, što se može vidjeti iz priložene bibliografije. U nastavku je dato obrazloženje ključnih rezultata publikovanih u renomiranim međunarodnim naučnim časopisima, koji predstavljaju glavne doprinose doktorske disertacije.

U naučnom radu „ *Distributed Observer Approach to Cooperative Output Regulation of Multi-Agent Systems Without Exchange of Controller States*", koji je publikovan u časopisu *IEEE Access* sa IF-om 3.9, prezentovani su rezultati istraživanja čiji su osnovni ciljevi bili razvoj novih upravljačkih protokola za kooperativnu regulaciju izlaza (eng. *cooperative output regulation*, COR) u heterogenim multiagentnim sistemima, kao i razvoj odgovarajuće metodologije za sintezu parametara razvijenih protokola. Za razliku od većine radova u literaturi koji su fokusirani isključivo na mreže neintrospektivnih ili na mreže introspektivnih agenata, u sprovedenim istraživanjima neintrospektivni i introspektivni agenti su tretirani u jedinstvenom teorijskom okviru koji je utemeljen na principima $H_\infty$ teorije upravljanja. U radu su predložena dva originalna upravljačka protokola – jedan za rješavanje COR problema u mrežama introspektivnih agenata, i drugi za rješavanje COR problema u mrežama neintrospektivnih agenata. Oba protokola su zasnovana na distribuiranom opserveru i zahtijevaju da agenti međusobno razmjenjuju samo mjerenja izlaza, čime se značajno smanjuje komunikaciono opterećenje. Štaviše, u scenarijima u kojima agenti mogu da mjere relativne izlaze jedni u odnosu na druge, predložene protokole je moguće implementirati bez uspostavljanja komunikacije između agenata. U okviru sprovedenih istraživanja razvijena je i metodologija za sintezu parametara predloženih protokola. U odnosu na postojeća rješenja predloženi protokoli garantuju rješivost COR problema za širu klasu sistema, dok se primjenom predloženog metoda za sintezu parametara postižu bolje performanse spregnutog sistema u odnosu na postojeće *low-gain* metode. Konkretno, sprovedena je detaljna analiza i dokazano je da rješivost COR problema unaprijed može biti zagarantovana za: introspektivne agente sa opštom linearnom dinamikom i neintrospektivne agente koji su stabilni u otvorenoj sprezi, a sve pod pretpostavkom da referentni signali i poremećaji ne rastu eksponencijalno. U radu su prikazani i rezultati numeričkih simulacija u kojima su verifikovani teorijski rezultati i demonstrirana efikasnost predložene metodologije za sintezu kontrolera. Takođe je izvršena komparacija sa postojećim rješenjima iz literature. Rezultati istraživanja koji su publikovani u ovom radu su prezentovani u III i V glavi disertacije.

Naučni rad „ *Output containment control in heterogeneous multi-agent systems without exchange of controller states*" publikovan je u časopisu *European Journal of Control* čiji je IF 3.4. U ovom radu su prezentovani rezultati istraživanja čiji je cilj bio razvoj novog upravljačkog protokola za *output containment control* (OCC) u heterogenim multiagentnim sistemima u prisustvu eksternih poremećaja, pod pretpostavkom da su samo izlazi susjednih agenata dostupni za sintezu lokalnih kontrolera. Na ovaj način se postiže smanjenje ili potpuna eliminacija komunikacije između agenata, a sve u zavisnosti od tipa senzora koji se koriste u praktičnoj implementaciji. OCC protokol koji je predložen koristi distribuirani opserver za estimaciju konveksne kombinacije stanja lidera, dok se na nivou pojedinačnog agenta koristi lokalni opserver za estimaciju eksternih poremećaja.

Takođe, razvijen je novi metod za sintezu parametara kontrolera koji se zasniva na konceptima iz $H_\infty$ teorije i algebarskim Rikatijevim jednačinama, i koji omogućava adekvatno podešavanje performansi spregnutog sistema. Teorijske garancije za rješivost OCC problema predloženim protokolom su uspostavljene u dva slučaja: kada je dinamika pratilaca minimalno fazna i desno-invertabiln i za egzosisteme sa polovima na imaginarnoj osi, dok pratioci mogu imati proizvoljnu linearnu dinamiku. Prezentovani rezultati numeričkih simulacija potvrđuju teorijske zaključke i demonstriraju efikasnost predloženog kontrolera i postupka za njegovu sintezu. Rezultati istraživanja koji su publikovani u ovom radu su prezentovani u IV i V glavi disertacije.

U naučnom radu "*Cooperative tracking control of single-integrator multi-agent systems with multiple leaders*", koji je objavljen u časopisu *European Journal of Control* sa IF-om 3.4, predložen je distribuirani algoritam za kooperativno praćenje trajektorija u multiagentnim sistemima sa više lidera, pri čemu je dinamika pratilaca opisana jednostrukim integratorom. Predložene su dvije varijante algoritma - za introspektivne i neintrospektivne agente. U drugom slučaju, pratioci posredstvom distribuiranog opservera i relativnih mjerenja vrše estimaciju sopstvenog stanja u apsolutnom koordinatnom sistemu. U radu su izvedeni lokalni uslovi za stabilnost čija ispunjenost garantuje da će greška u praćenju referentnih trajektorija konvergirati ka nuli. Takođe su predloženi odgovarajući metodi za sintezu parametara kontrolera i opservera u cilju zadovoljenja ovih uslova. U prvoj glavi disertacije predstavljen je problem kooperativnog praćenja trajektorija u multiagentnim sistemima, gdje je i naveden doprinos ovog istraživanja.

Jedan dio rezultata istraživanja kandidata publikovan je u četiri rada prezentovana na međunarodnim konferencijama koje su indeksirane u IEEE Xplore i Scopus bazama.

| **Datum i ovjera (pečat i potpis odgovorne osobe)** | |
|---|---|
| U Podgorici,<br>22.04.2024. | MP UNIVERZITET CRNE GORE UCG Elektrotehnički fakultet<br>DEKAN |

**Prilog dokumenta sadrži:**

1. Potvrdu o predaji doktorske disertacije organizacionoj jedinici
2. Odluku o imenovanju komisije za pregled i ocjenu doktorske disertacije
3. Kopiju rada publikovanog u časopisu sa odgovarajuće liste
4. Biografiju i bibliografiju kandidata
5. Biografiju i bibliografiju članova komisije za pregled i ocjenu doktorske disertacije sa potvrdom o izboru u odgovarajuće akademsko zvanje i potvrdom da barem jedan član komisije nije u radnom odnosu na Univerzitetu Crne Gore

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Broj: 02/1-544/2
Datum: 18.03.2024.

Na osnovu službene evidencije i dokumentacije Elektrotehničkog fakulteta u Podgorici, izdaje se

# POTVRDA

MSc **Luka Martinović**, student doktorskih studija na Elektrotehničkom fakultetu u Podgorici, dana 18.03.2024. godine dostavio je ovom Fakultetu doktorsku disertaciju pod nazivom: **„Kooperativno upravljanje heterogenim multiagentnim sistemima bez razmjene stanja kontrolera“**, na dalje postupanje.

**SEKRETAR**

**mr Valentina Lješević - Dedić**

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Na osnovu člana 64 Statuta Univerziteta Crne Gore, u vezi sa članom 41 Pravila doktorskih studija, na predlog Komisije za doktorske studije, Vijeće Elektrotehničkog fakulteta u Podgorici, na sjednici od 18.04.2024. godine, donijelo je

**ODLUKU**

Predlaže se Komisija za ocjenu doktorske disertacije **„Kooperativno upravljanje heterogenim multiagentnim sistemima bez razmjene stanja kontrolera"** kandidata MSc **Luke Martinovića,** u sastavu:

1. Dr Željko Đurović, redovni profesor Elektrotehničkog fakulteta Univerziteta u Beogradu, predsjednik,
2. Dr Božo Krstajić, redovni profesor Elektrotehničkog fakulteta Univerziteta Crne Gore, mentor,
3. Dr Žarko Zečević, vanredni profesor Elektrotehničkog fakulteta Univerziteta Crne Gore, član.

**-VIJEĆE ELEKTROTEHNIČKOG FAKULTETA-**

**Vršilac funkcije DEKANa**
Lutovac
**Prof. dr Budimir Lutovac**

Univerzitet Crne Gore Elektrotehnički fakultet

Dostavljeno:
- Senatu,
- Odboru za doktorske studije,
- u dosije,
- a/a.

**Spisak radova sa rezultatima iz doktorske disertacije:**

- **Naučni časopisi na SCI/SCIE listi:**

1) Luka Martinović, Žarko Zečević, Božo Krstajić, Output containment control in heterogeneous multi-agent systems without exchange of controller states, European Journal of Control, Volume 75, 2024, 100889, ISSN 0947-3580, https://doi.org/10.1016/j.ejcon.2023.100889.
2) L. Martinović, Ž. Zečević and B. Krstajić, "Distributed Observer Approach to Cooperative Output Regulation of Multi-Agent Systems Without Exchange of Controller States," in IEEE Access, vol. 11, pp. 81419-81433, 2023, doi: 10.1109/ACCESS.2023.3300806.
3) Luka Martinović, Žarko Zečević, Božo Krstajić, Cooperative tracking control of single-integrator multi-agent systems with multiple leaders, European Journal of Control, Volume 63, 2022, Pages 232-239, ISSN 0947-3580, https://doi.org/10.1016/j.ejcon.2021.11.003.

- Naučne konferencije:

1) L. Martinović, Ž. Zečević and B. Krstajić, "Regulated Output Synchronization of Multi-Agent Systems via Output Feedback," 2022 26th International Conference on Information Technology (IT), Zabljak, Montenegro, 2022, pp. 1-4, doi: 10.1109/IT54280.2022.9743524.
2) L. Martinović, Z. Zečević and B. Krstajić, "Cooperative Tracking Control of Multi-Agent Systems with General Linear Dynamics," 2021 25th International Conference on Information Technology (IT), Zabljak, Montenegro, 2021, pp. 1-4, doi: 10.1109/IT51528.2021.9390134.
3) L. Martinović, Ž. Zečević and B. Krstajić, "Cooperative Output Regulation of Multi-Agent Systems with Single-Integrator Dynamics," 2020 28th Telecommunications Forum (TELFOR), Belgrade, Serbia, 2020, pp. 1-4, doi: 10.1109/TELFOR51502.2020.9306565.
4) L. Martinović, Ž. Zečević and B. Krstajić, "Distributed Control Strategy for Multi-Agent Systems Using Consensus Among Followers," 2020 24th International Conference on Information Technology (IT), Zabljak, Montenegro, 2020, pp. 1-4, doi: 10.1109/IT48810.2020.9070617.

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# Output containment control in heterogeneous multi-agent systems without exchange of controller states

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ABSTRACT

In this paper, we propose an observer-based distributed protocol for solving the output containment problem in linear heterogeneous multi-agent systems under the external disturbances. The distinct feature of the proposed protocol is that it does not require exchange of the internal controller states, which significantly reduces or completely eliminates communication burden. A sufficient local stability condition is derived and a novel controller design procedure is proposed based on tools from $\mathcal{H}_\infty$ theory and algebraic Riccati equations. An extensive solvability analysis of the output containment control problem under the proposed protocol is carried out and theoretical guarantees for the existence of a solution for a wide class of agents' dynamics and graph topologies are established. Numerical simulations that demonstrate the effectiveness of the proposed protocol are performed.



## 1. Introduction

During the past two decades, there has been a tremendous surge of interest in the area of cooperative control of multi-agent systems (MASs) due to its broad applications in multi-robot systems, sensor networks, smart grids, and so forth [2,24,30,44].

The problem of achieving consensus has particularly stood out as a fundamental problem in cooperative control. Consensus problems can be divided into two classes, depending on whether there is a leader present in the network. In the leaderless consensus, the goal for the agents is to reach an agreement regarding a certain quantity of interest that depends on the state/output of all agents [14,28,29]. On the other hand, in leader-following consensus, the goal is to synchronize the follower agents to a trajectory generated by the leader. Typically, a distributed observer is designed to estimate the leader's states, and then these estimated states are utilized in the control law of each follower [20]. Recent research efforts have focused on developing distributed observers with specific features such as finite-time convergence [36], resilience to DoS attacks [46], and the ability to handle communication delays [11] or actuator faults [4]. Furthermore, there has been an increased interest in studying consensus over signed networks, which provide a framework for modeling both cooperative and antagonistic interactions between agents [22,26].

When multiple leaders are present, often the main objective is to drive the states/outputs of each follower into the convex hull spanned by the leaders' trajectories. In the literature, this problem is known as the *containment control* problem. The containment control problem has garnered significant interest from researchers due to its immense potential for practical applications. A classic example is that of vehicles moving through environments that contain hazardous areas, with only leaders being equipped with sensors for detecting those areas. The goal of the leaders is to generate proper trajectories in order to steer all agents away from these areas [1]. Many results on the state containment control of homogeneous linear networks have been reported in the recent years [8,23,40,42]. A fully distributed protocol for solving the containment control problem in homogeneous linear MASs subject to DoS attacks was proposed in [23]. The state containment for MASs subject to known exogenous disturbances is considered in [8,42], while agents with unknown dynamics are recently investigated in [40].

In the case of heterogeneous MASs with general linear dynamics, it is often required that followers' outputs are contained within a convex hull formed by the leaders' outputs, since the internal states of the agents may not be directly comparable. In [45], the output containment problem has been solved by a distributed internal model-based protocol, while the approaches presented in [3,7,12] are based on a distributed observer. An adaptive distributed observer for solving the optimal output containment problem when leader's model is not known to all followers is introduced in [25]. Other results include event-triggered controllers [32], reinforcement learning-based controllers [37], attack-resilient

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controllers [31], etc. In contrast to the containment control in homogeneous networks, there have been only a few publications dealing with the exogenous disturbances acting on the heterogeneous followers. A protocol that addresses the output containment in heterogeneous MASs subject to unknown external disturbances with optimized $\mathcal{H}_\infty$ disturbance attenuation level has been proposed in [39]. On the other hand, in [15,16] the authors designed a distributed observer for rejection of the known disturbances using an output regulation framework. However, the disturbance frequencies are assumed to be the same as those of a reference signal.

In cooperative control protocols [3,4,7,11,12,15,16,20,31,36,37,39], the internal controller states are transmitted between agents via the communication network and used as input for the distributed observer. Some protocols, such as those in [25,32,46], require the additional exchange of output measurements. Significant research efforts have been devoted to developing consensus and containment protocols that rely solely on the exchange of output measurements. The advantages of such types of protocols are at least twofold. Firstly, since the outputs are typically of lower dimensions compared to the controller states, they can greatly reduce the communication burden [18,19]. Secondly, in scenarios where agents can measure the relative output information of their neighbors, these protocols can be implemented without the need for establishing a communication network, making them resilient to network attacks [21,43]. However, in this communication-limited setting, where agents lack information about their neighboring agents' controller states, the design of distributed observers becomes particularly challenging. In [6], the output consensus problem without the exchange of controller states has been addressed for networks of identical minimum-phase agents, while networks of non-identical agents with right-invertible dynamics are investigated in [41]. The tracking problem in homogeneous MASs with general linear dynamics is resolved in [21,43], where the authors introduce a local observer that estimates synchronization error. In [18,19], a low-gain technique has been developed based on the small gain condition, aimed at designing a distributed observer for heterogeneous MASs with general linear dynamics. However, this method does not guarantee the solvability of the consensus problem in the presence of external disturbances, thereby limiting its practicality. Furthermore, in [34] the containment control problem is addressed for homogeneous MASs without requiring agents to share their internal controller states. To the best of the author's knowledge, such types of observer-based containment protocols are currently lacking for heterogeneous MASs.

Motivated by the aforementioned discussion, in this paper we propose a novel distributed observer-based protocol that solves output containment problem in linear heterogeneous MASs. The proposed protocol does not require exchange of the internal controller states and is capable of rejecting exogenous disturbances. The external disturbances are estimated by the means of a local observer and compensated in the control input. The interaction topology is directed, and the only requirement is that there exists a directed path from at least one leader to each follower in the network.

The contributions of this paper can be summarized as follows:

1. In contrast to the observer-based consensus [4,11,20,36,46] and containment protocols [3,7,12,15,16,25,31,32,37,39], the protocol proposed in this paper does not require agents to share the internal controller states with each other. Instead, only output information is transmitted between the agents, thus reducing the communication burden. Moreover, if the followers are able to measure relative information of the neighboring agents, then the proposed protocol is attack-free [21].
2. Unlike the protocols [3,7,25,31,32,37,39], the proposed protocol solves the output containment problem in the presence of global disturbances. Since the presence of such disturbances is unavoidable in practical scenarios, this makes the proposed protocol paper more applicable. On the other hand, compared to [15,16], the approach taken in this paper is more general, since the global disturbances and the reference signal considered in this paper are generated by the separate exosystems.
3. A novel method for calculating the controller parameters, which is based on tools from $\mathcal{H}_\infty$ theory and algebraic Riccati equation (ARE) methods, is developed. Compared to the low-gain method [18,19], the proposed approach offers greater flexibility in designing the controller parameters, allowing for better tuning of the system performance. Furthermore, theoretical guarantees for the existence of such controller are established in two cases: a) when agents are minimum-phase and right-invertible, and b) when the poles of the exosystem are on the imaginary axis, while agents may have arbitrary linear dynamics.
4. In comparison to the protocol [34] proposed for homogeneous agents, in this paper the agents are allowed to be heterogeneous with general linear dynamics. Furthermore, the proposed methodology can also be utilized to address the leader-following consensus problem without requiring agents to share their internal controller states, given that this problem represents a special case of containment control. Moreover, compared to the consensus protocols [6,18,19,21,41,43], which also rely solely on the exchange of output measurements, our approach can accommodate a broader range of agents' dynamics.

The paper is organized in the following way. Preliminaries on graph theory and statement of the output containment control objective are given in Section 2. A novel output containment control protocol is proposed in Section 3, where the explicit conditions for solving the considered problem are given. Moreover, a design procedure for determining the controller parameters is presented. The theoretical guarantees for the existence of a solution are derived in Section 4. Section 5 contains numerical simulations which verify the obtained results. Finally, in Section 6 we give concluding remarks.

*Notation:* $\lambda(A)$ represents the spectrum, while $\rho(A)$ is the spectral radius of the square matrix $A$. $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix, $1_n \in \mathbb{R}^n$ is a vector with all entries of one and $0_n \in \mathbb{R}^n$ is a vector of all zeroes. If there is no confusion about the dimension, the subscript will be dropped. Kronecker product is denoted by $\otimes$. The operator diag$\{\cdot\}$ builds a (block) diagonal matrix from its arguments. The standard $\mathcal{H}_\infty$ norm is denoted as $\|\cdot\|_\infty$. For a symmetric matrix $P$, we write $P > 0$ ($P < 0$) if it is positive (negative) definite.

## 2. Preliminaries

### 2.1. Graph theory

Consider a group of $N+M$ interconnected agents, whose interaction topology is represented by a directed graph (digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. We index the agents such that the agents $\mathbb{F} = \{1, 2, \ldots, N\}$ are the followers and $\mathbb{L} = \{N+1, N+2, \ldots, N+M\}$ are the leaders. The set of nodes is defined as $\mathcal{V} = \{\nu_1, \nu_2, \ldots, \nu_{N+M}\}$, while the set of edges is denoted as $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. The digraph is time-invariant and can be described by the corresponding adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(N+M)\times(N+M)}$, where $a_{ij} > 0$ if $(\nu_j, \nu_i) \in \mathcal{E}$, otherwise $a_{ij} = 0$. We assume that the digraph is simple, i.e. there are no repeated edges or self loops ($a_{ii} = 0,\ \forall i$). An edge from $j$ to

$i$, denoted by $(v_j, v_i) \in \mathcal{E}$, means that the node $v_i$ receives the information from the node $v_j$, $j \neq i$. In this situation, node $v_j$ is called a neighbor of node $v_i$. The set of neighbors of node $v_i$ is denoted as $\mathcal{N}_i = \{j|(v_j, v_i) \in \mathcal{E}\}$. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{(N+M)\times(N+M)}$ is then defined as $l_{ij} = \sum_{k=1}^{N+M} a_{ik}$ for $j = i$ and $l_{ij} = -a_{ij}$ for $j \neq i$. We assume that the leaders do not receive any information from the followers, which leads to the partition of Laplacian matrix

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_1 & \mathcal{L}_2 \\ 0_{M\times N} & 0_{M\times M} \end{bmatrix},$$

where $\mathcal{L}_1 \in \mathbb{R}^{N\times N}$ and $\mathcal{L}_2 \in \mathbb{R}^{N\times M}$.

A sequence of the successive edges $\{(v_i, v_l), (v_l, v_k), \dots, (v_p, v_j)\}$ is called a directed path from node $i$ to $j$. A directed path whose first and last node are the same, i.e. $v_i = v_j$, is called a cycle. The digraph which contains no cycles is called acyclic. A digraph $\mathcal{G}$ is said to contain *a directed spanning tree* if there exists at least one node having a directed path to any of the other nodes, while it is said to have *a united spanning tree* if for each follower $i \in \mathbb{F}$, there exists at least one leader $k \in \mathbb{L}$ that has a directed path to it.

Without loss of generality, it will be assumed that the adjacency and Laplacian matrices of the digraph are normalized. That is, for $i \in \mathbb{F}$ we have $\sum_{j\in\mathcal{N}_i} a_{ij} = 1$.

**Lemma 1** [1]. *If $\mathcal{G}$ has a united spanning tree, then the matrix $-\mathcal{L}_1^{-1}\mathcal{L}_2$ is non negative and $-\mathcal{L}_1^{-1}\mathcal{L}_2 1_M = 1_N$.*

**Lemma 2.** *The inequality $\rho(I - \mathcal{L}_1) < 1$ holds if and only if the digraph $\mathcal{G}$ contains a united spanning tree.*

**Proof.** Introduce the new Laplacian matrix

$$\bar{\mathcal{L}} = \begin{bmatrix} \mathcal{L}_1 & \mathcal{L}_2 1_M \\ 0_{1\times N} & 0 \end{bmatrix}.$$

The digraph associated with $\bar{\mathcal{L}}$ has a directed spanning tree with the leader as the root if and only if $\mathcal{G}$ has a united spanning tree [1]. The non negativity of matrix $\bar{A}_1 = I - \mathcal{L}_1$ allows Perron-Frobenius theorem to be applied, which implies that $\rho(\bar{A}_1)$ is its eigenvalue. Since $\bar{A} = \begin{bmatrix} \bar{A}_1 & -\mathcal{L}_2 1_M \\ 0 & 1 \end{bmatrix}$, it is clear that it contains an eigenvalue 1 in addition to eigenvalues of $\bar{A}_1$. The matrix $\bar{A}$ is row-stochastic, thus according to Huang and Ye [10] it has a simple eigenvalue $\rho(\bar{A}) = 1$ if and only if the digraph $\bar{\mathcal{G}}$ contains a directed spanning tree with the leader as a root. Thus, it can be concluded that $\rho(\bar{A}_1) < 1$, which completes the proof. □

### 2.2. Multi-agent systems dynamics

The followers are assumed to be heterogeneous and have the general linear dynamics

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i + E_i d \\ y_i = C_i x_i + Q_i d \end{cases}, \ i \in \mathbb{F}, \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $y_i \in \mathbb{R}^p$ are the state, input and output of the $i$th follower, respectively. The disturbance is generated by an autonomous exosystem modeled as

$$\dot{d} = Dd, \tag{2}$$

where $d \in \mathbb{R}^q$. The leaders have identical models and the dynamics of the $k$th leader is given by

$$\begin{cases} \dot{\zeta}_k = S\zeta_k \\ y_k = R\zeta_k \end{cases}, \ k \in \mathbb{L}, \tag{3}$$

where $\zeta_k \in \mathbb{R}^r$, $y_k \in \mathbb{R}^p$.

**Remark 1.** The presence of external disturbances is inevitable in practical multi-agent systems. An example would be a network of moving UAVs affected by strong winds whose dynamics can be described by an exosystem. In fact, several authors have adopted the disturbance model as (2) [15,16]. It should be noted, that in our approach the disturbances are estimated at the local level, and therefore different disturbance matrices $D_i$ can be adopted for each agent [33]. For example, the matrix $D_i$ can incorporate frequencies that are characteristic of both the global and local environment. It is important to highlight that the controller design procedure remains unchanged, regardless of whether the matrix $D$ or individual matrices $D_i$ are adopted.

The following assumptions are made on the network topology and agents' dynamics.

**Assumption 1.** The digraph $\mathcal{G}$ contains a united spanning tree.

**Assumption 2.** The matrix $S$ has no strictly stable poles, i.e. $\lambda(S) \in \bar{\mathbb{C}}^+$.

**Assumption 3.** The matrix $D$ has no strictly stable poles, i.e. $\lambda(D) \in \bar{\mathbb{C}}^+$.

**Assumption 4.** The pairs $(A_i, B_i)$ are stabilizable for all $i \in \mathbb{F}$.

**Assumption 5.** The pairs $\left(\begin{bmatrix} C_i & Q_i \end{bmatrix}, \begin{bmatrix} A_i & E_i \\ 0 & D \end{bmatrix}\right)$, $i \in \mathbb{F}$, and $(R, S)$ are detectable.

**Assumption 6.** The linear matrix equations

$$\begin{cases} \Pi_i^\zeta S = A_i \Pi_i^\zeta + B_i \Gamma_i^\zeta \\ 0 = C_i \Pi_i^\zeta - R \end{cases}, \tag{4a}$$

$$\begin{cases} \Pi_i^d D = A_i \Pi_i^d + B_i \Gamma_i^d + E_i \\ 0 = C_i \Pi_i^d + Q_i \end{cases} \tag{4b}$$

have a unique solution pairs $(\Pi_i^\zeta, \Gamma_i^\zeta)$ and $(\Pi_i^d, \Gamma_i^d)$ for $i \in \mathbb{F}$, respectively.

**Remark 2.** The assumptions made in this paper are common in the literature of the output containment control. Assumption 1 is necessary for solving the containment problem in a distributed way. Assumptions 2 and 3 are made to avoid trivial case of strictly stable $S$ and $D$, since the eigenvalues with negative real parts exponentially decay to zero and do not affect the asymptotic behavior of the closed-loop system. Moreover, if the output containment control problem is solved for a linear multi-agent system under Assumptions 2 and 3, then it is also solved when the assumptions are violated, as stated in [45]. Assumptions 4 and 5 are standard necessary stabilizability and detectability assumptions. It should be noted that when $Q_i = 0$, Assumption 5 reduces to the detectability of $(C_i, A_i)$ and $(E_i, A_i)$ [5]. Assumption 6 involves regulator equations derived from classical control theory. It should be noted that these equations can generally be solved when the number of inputs is equal to or greater than the number of outputs, with numerous practical models satisfying this assumption [4,5,12,20,34,38].

### 2.3. Output containment control objective

The distance from $x \in \mathbb{R}^n$ to the set $\mathcal{C} \in \mathbb{R}^n$ in the sense of Euclidean norm is denoted as $\text{dist}(x, \mathcal{C})$, i.e.

$$\text{dist}(x, \mathcal{C}) = \inf_{y\in\mathcal{C}} \|x - y\|_2. \tag{5}$$

**Definition 1.** A set $\mathcal{C} \subseteq \mathbb{R}^n$ is convex if $(1-\lambda)x + \lambda y \in \mathcal{C}$ for any $x, y \in \mathcal{C}$ and any $\lambda \in [0, 1]$. The convex hull $\text{Co}(X)$ of a finite set of points $X = \{x_1, x_2, \dots, x_q\}$ is the minimal convex set containing all points in $X$, i.e. $\text{Co}(X) = \{\sum_{i=1}^q \alpha_i x_i | x_i \in X, \alpha_i \in \mathbb{R}_{\geq 0}, \sum_{i=1}^q \alpha_i = 1\}$.

**Definition 2.** (*Output Containment Control Objective*) Consider the followers' dynamics (1) and the leaders' dynamics (3) on a fixed directed communication graph $\mathcal{G}$. The output containment control objective is to design a distributed control protocol such that for all initial states the overall closed-loop system is globally stable and the outputs of all followers converge to the convex hull spanned by the outputs of the dynamic leaders, that is

$$\lim_{t\to\infty} \text{dist}\big(y_i(t), \text{Co}\big(y_k(t), k\in\mathbb{L}\big)\big) = 0,\ i\in\mathbb{F}.$$

Define the local neighborhood output containment error of the $i$th follower as

$$e_i = \sum_{j=1}^{N+M} a_{ij}\big(y_i - y_j\big),\ i\in\mathbb{F}. \tag{6}$$

Then, the global output containment error can be written as

$$e = \big(\mathcal{L}_1\otimes I_p\big)y_F + \big(\mathcal{L}_2\otimes I_p\big)y_L, \tag{7}$$

where $e=[e_1^T, e_2^T,\dots,e_N^T]^T$, $y_L=[y_{N+1}^T, y_{N+2}^T,\dots,y_{N+M}^T]^T$ and $y_F=[y_1^T,y_2^T,\dots,y_N^T]^T$.

Consider the heterogeneous MAS (1), (3) subject to the disturbance (2). Then, we have the following lemma.

**Lemma 3.** *Suppose that Assumption 1 holds. Then, the output containment control objective is achieved if* $\lim_{t\to\infty} e(t)=0$.

**Proof.** Let $e=0$. Then, under Assumption 1 we get

$$y_F = -\big(\mathcal{L}_1^{-1}\mathcal{L}_2\otimes I_p\big)y_L,$$

where the property of Kronecker product $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$ was utilized. By the means of Lemma 1, we conclude that the term $-(\mathcal{L}_1^{-1}\mathcal{L}_2\otimes I_p)y_L$ is a column vector of the stacked followers' outputs $y_i,\ i\in\mathbb{F}$, where each $y_i$ is in the convex hull generated by the leaders' outputs. □

## 3. Main results

In this section, we propose a novel observer-based protocol for solving the output containment control problem that does not require an exchange of the controller states. The controller of each agent consists of: 1) the local observer for estimating the disturbance and states of the agent, 2) the distributed observer for estimating the convex combination of the leaders' states. The proposed protocol takes the following form:

$$\begin{aligned}\begin{bmatrix}\dot{\hat{x}}_i\\ \dot{\hat{d}}_i\end{bmatrix} &= \begin{bmatrix}A_i & E_i\\ 0 & D\end{bmatrix}\begin{bmatrix}\hat{x}_i\\ \hat{d}_i\end{bmatrix} + \begin{bmatrix}B_i\\ 0\end{bmatrix}u_i + \begin{bmatrix}L_i^x\\ L_i^d\end{bmatrix}\left(y_i - \begin{bmatrix}C_i & Q_i\end{bmatrix}\begin{bmatrix}\hat{x}_i\\ \hat{d}_i\end{bmatrix}\right)\\ \dot{\hat{\zeta}}_i &= S\hat{\zeta}_i + L_i^\zeta\varepsilon_i\\ u_i &= K_i^x\hat{x}_i + K_i^d\hat{d}_i + K_i^\zeta\hat{\zeta}_i\end{aligned} \tag{8}$$

where $\hat{x}_i\in\mathbb{R}^{n_i}$, $\hat{d}_i\in\mathbb{R}^q$ and $\hat{\zeta}_i\in\mathbb{R}^r$ represent the estimates of the agents states, disturbance generating exosystem states and convex combination of leaders states, respectively. The controller gains $K_i^x$, $K_i^d$, $K_i^\zeta$ and the observer gains $L_i^x$, $L_i^d$, $L_i^\zeta$ are the parameters of the appropriate dimension that need to be designed. The virtual error signal is defined as

$$\varepsilon_i = e_i - y_i + R\hat{\zeta}_i. \tag{9}$$

**Remark 3.** It should be noted that the existing observer-based output containment protocols [3,7,12,15,16,25,31,32,37,39] require an exchange of the internal controller states between the neighbouring nodes. The salient feature of the protocol (8) is that it only requires exchange of the output measurements, which are typically of a lower dimension than the controller states, thus reducing communication burden. Furthermore, in scenarios where the relative information of neighboring agents is measurable, the proposed protocol can be implemented without establishing communication among agents. In this setting, the protocol is inherently secure and immune to attacks [21,43]. For instance, in multi-robot systems, cameras can be utilized to obtain the relative positions of the robots with respect to each other [18]. However, if $i$th agent can only measure $y_i$, the output measurement needs to be transmitted to neighboring agents through the communication network. In such cases, additional protection mechanisms, such as the one described in [22], must be employed to prevent potential attacks. Compared to [3,7,25,31,32,37,39], another favorable property of the proposed protocol lies in its ability to handle external disturbances. Furthermore, the proposed protocol is more general than those in [15,16], where it is assumed that the disturbance and reference signals are generated by the same exosystem.

The closed-loop dynamics for each agent in the network can be obtained by substituting (8) into (1). Introduce $x=[x_1^T,x_2^T,\dots,x_N^T]^T$, $\underline{d}=1_N\otimes d$, $\zeta=[\zeta_{N+1}^T,\zeta_{N+2}^T,\dots,\zeta_{N+M}^T]^T$, $\hat{x}=[\hat{x}_1^T,\hat{x}_2^T,\dots,\hat{x}_N^T]^T$, $\hat{d}=[\hat{d}_1^T,\hat{d}_2^T,\dots,\hat{d}_N^T]^T$ and $\hat{\zeta}=[\hat{\zeta}_1^T,\hat{\zeta}_2^T,\dots,\hat{\zeta}_N^T]^T$. Define $x_c=[x^T,\hat{x}^T,\hat{d}^T,\hat{\zeta}^T]^T$. Then, the closed-loop dynamics can be written in a global form as

$$\begin{cases}\dot{x}_c = A_cx_c + B_c^d\underline{d} + B_c^\zeta\zeta\\ e = C_cx_c + D_c^d\underline{d} + D_c^\zeta\zeta\end{cases} \tag{10}$$

with the closed-loop matrices being equal to

$$A_c = \begin{bmatrix} A & BK^x & BK^d & BK^\zeta\\ L^xC & A+BK^x-L^xC & E+BK^d-L^xQ & BK^\zeta\\ L^dC & -L^dC & \tilde{D}-L^dQ & 0\\ L^\zeta(\tilde{\mathcal{L}}_1-I)C & 0 & 0 & \tilde{S}+L^\zeta\tilde{R}\end{bmatrix},$$

$$B_c^d = \begin{bmatrix}E\\ L^xQ\\ L^dQ\\ L^\zeta(\tilde{\mathcal{L}}_1-I)Q\end{bmatrix},\quad B_c^\zeta = \begin{bmatrix}0\\0\\0\\ L^\zeta\tilde{\mathcal{L}}_2(I_M\otimes R)\end{bmatrix},$$

$$C_c = \begin{bmatrix}\tilde{\mathcal{L}}_1C & 0 & 0 & 0\end{bmatrix},\ D_c^d = \tilde{\mathcal{L}}_1Q,\ D_c^\zeta = \tilde{\mathcal{L}}_2(I_M\otimes R),$$

where $\tilde{\mathcal{L}}_1=\mathcal{L}_1\otimes I_p$, $\tilde{\mathcal{L}}_2=\mathcal{L}_2\otimes I_p$, $\tilde{S}=I_N\otimes S$, $\tilde{D}=I_N\otimes D$, $\tilde{R}=I_N\otimes R$. Note that we have adopted the notation $\Phi=\text{diag}\{\Phi_i\}$, where $\Phi=(A,B,C,E,Q,K^x,K^d,K^\zeta,L^x,L^d,L^\zeta,\Pi^d,\Pi^\zeta)$.

### 3.1. Output containment control

In this subsection, it will be shown that under Assumptions 1–3 and 6, there exists a solution to the output containment problem and that achieving the output containment objective is equivalent to ensuring that $A_c$ is a Hurwitz stable matrix. The problem of selecting the controller gains such that $A_c$ is Hurwitz will be the subject of the remaining subsections.

**Lemma 4.** *Let $A_c$ be a Hurwitz stable matrix. If there exist matrices $X^d$ and $X^\zeta$ such that the following equations hold*

$$\begin{cases}X^d\tilde{D} = A_cX^d + B_c^d\\ 0 = C_cX^d + D_c^d\end{cases} \tag{11a}$$

$$\begin{cases}X^\zeta(I_M\otimes S) = A_cX^\zeta + B_c^\zeta\\ 0 = C_cX^\zeta + D_c^\zeta\end{cases} \tag{11b}$$

*then for any $\underline{d}(t)$ and $\zeta(t)$ it follows that* $\lim_{t\to\infty}e(t)=0$.

**Proof.** Introduce $\bar{x}=x_c-X^d\underline{d}-X^\zeta\zeta$. Substituting $\bar{x}$ into (10) gives

$$\dot{\bar{x}} = A_c\bar{x} + (A_cX^d + B_c^d - X^d(I_N\otimes D))\underline{d}$$

$$+\left(A_cX^{\zeta}-B_c^{\zeta}-X^{\zeta}(I_M\otimes S)\right)\zeta$$
$$e=C_c\tilde{x}+(C_cX^d+D_c^d)\underline{d}+(C_cX^{\zeta}+D_c^{\zeta})\zeta.$$

If (11) holds, then we have

$$\begin{cases}\dot{\tilde{x}}=A_c\tilde{x}\\ e=C_c\tilde{x}\end{cases} \tag{12}$$

and since $A_c$ is Hurwitz, $\tilde{x}\to 0$ as $t\to\infty$, which further implies $\lim_{t\to\infty}e(t)=0$. □

**Remark 4.** The existence of a solution pair $(X^d,X^{\zeta})$ to (11) is a necessary but not a sufficient condition for achieving the output containment objective, as it can be deduced from Lemma 3. In order to solve the output containment problem, Assumption 1 must also hold [Theorem 3.1, [1]].

From now on let the following relations hold

$$\Gamma_i^{\zeta}=K_i^x\Pi_i^{\zeta}+K_i^{\zeta},\;\Gamma_i^d=K_i^x\Pi_i^d+K_i^d,\;i\in\mathbb{F}, \tag{13}$$

where $(\Pi_i^{\zeta},\Gamma_i^{\zeta})$ and $(\Pi_i^d,\Gamma_i^d)$ are the solution pairs of (4).

**Theorem 1.** *Let $A_c$ be Hurwitz stable matrix and suppose that Assumptions 1–3 and 6 hold. Then, the output containment control objective is achieved by MAS (1)–(3) under protocol (8).*

**Proof.** With Lemma 4 in mind, under Assumption 1, it is sufficient to show that a solution pair $(X^d,X^{\zeta})$ to (11) exists. Taking into account (13), regulator Eq. (4) can be rewritten in the global form as

$$\begin{cases}\Pi^d\tilde{D}=(A+BK^x)\Pi^d+BK^d+E\\ -Q=C\Pi^d\end{cases} \tag{14a}$$

$$\begin{cases}\Pi^{\zeta}\tilde{S}=(A+BK^x)\Pi^{\zeta}+BK^{\zeta}\\ \tilde{R}=C\Pi^{\zeta}\end{cases} \tag{14b}$$

Since $A_c$ is Hurwitz, the Sylvester Eq. (11) have a unique solution pair $(X^d,X^{\zeta})$ due to Assumptions 2 and 3. The solution pair is given by

$$X^d=\begin{bmatrix}\Pi^d\\ \Pi^d\\ I\\ 0\end{bmatrix},\;X^{\zeta}=\begin{bmatrix}-\Pi^{\zeta}\left(\mathcal{L}_1^{-1}\mathcal{L}_2\otimes I_r\right)\\ -\Pi^{\zeta}\left(\mathcal{L}_1^{-1}\mathcal{L}_2\otimes I_r\right)\\ 0\\ -\left(\mathcal{L}_1^{-1}\mathcal{L}_2\otimes I_r\right)\end{bmatrix}, \tag{15}$$

where $(\Pi^d,\Pi^{\zeta})$ are the solutions of (14), whose existence is ensured by Assumption 6. This can be easily checked by substituting (15) into (11), with taking into account (14). Note that the equalities $(I_N\otimes S)(\mathcal{L}_1^{-1}\mathcal{L}_2\otimes I_r)=(\mathcal{L}_1^{-1}\mathcal{L}_2\otimes S)=(\mathcal{L}_1^{-1}\mathcal{L}_2\otimes I_r)(I_M\otimes S)$ and similarly $(I_N\otimes R)(\mathcal{L}_1{}^{-1}\mathcal{L}_2\otimes I_p)=(\mathcal{L}_1{}^{-1}\mathcal{L}_2\otimes I_p)(I_M\otimes R)$ have been utilized. □

**Remark 5.** Since $x_c\to X^d\underline{d}+X^{\zeta}\zeta$ as $t\to\infty$, from (15) it can be concluded that $\hat{\zeta}\to-(\mathcal{L}_1^{-1}\mathcal{L}_2\otimes I_r)\zeta$. Therefore, the distributed observer with the state $\hat{\zeta}_i$ can be regarded as an estimator of the convex combination of the leaders' states, as previously stated. The convex combination is entirely determined by the network topology. Furthermore, the states of the local observers, i.e. $\hat{x}_i$ and $\hat{d}_i$, asymptotically converge to the true values of $x_i$ and $d$, respectively. Finally, the value of the follower's states in steady-state is determined by both network topology and solutions to the regulator Eq. (4).

### 3.2. Stability analysis

This subsection is concerned with deriving local stability conditions which, when satisfied for each agent, guarantee the stability of matrix $A_c$.

Define

$$H=\begin{bmatrix}A & E\\ 0 & \tilde{D}\end{bmatrix},\;L^{xd}=\begin{bmatrix}L^x\\ L^d\end{bmatrix},\;G=-\begin{bmatrix}C & Q\end{bmatrix}, \tag{16}$$

$$\tilde{A}=\begin{bmatrix}A+BK^x & BK^{\zeta}\\ L^{\zeta}(\tilde{\mathcal{L}}_1-I)C & \tilde{S}+L^{\zeta}\tilde{R}\end{bmatrix}. \tag{17}$$

**Lemma 5.** *The closed-loop matrix $A_c$ is Hurwitz stable if and only if matrices $H+L^{xd}G$ and $\tilde{A}$ are Hurwitz stable.*

**Proof.** Consider matrix $\tilde{A}_c$, obtained by the similarity transformation $\tilde{A}_c=T^{-1}A_cT$, where

$$T=\begin{bmatrix}I & 0 & 0 & 0\\ I & I & 0 & 0\\ 0 & 0 & I & 0\\ 0 & 0 & 0 & I\end{bmatrix}.$$

By applying the transformation we get

$$\tilde{A}_c=\begin{bmatrix}A+BK^x & BK^x & BK^d & BK^{\zeta}\\ 0 & A-L^xC & E-L^xQ & 0\\ 0 & -L^dC & \tilde{D}-L^dQ & 0\\ L^{\zeta}(\tilde{\mathcal{L}}_1-I)C & 0 & 0 & \tilde{S}+L^{\zeta}\tilde{R}\end{bmatrix}.$$

Rearrangement of the state coordinates leads to the similar matrix

$$\tilde{A}_c\sim\begin{bmatrix}H+L^{xd}G & 0\\ \star & \tilde{A}\end{bmatrix},$$

which by separation principle is Hurwitz stable if $\tilde{A}$ and $H+L^{xd}G$ are Hurwitz stable. □

In the following, we analyze stabilization properties of $H+L^{xd}G$ and $\tilde{A}$. Let

$$H_i=\begin{bmatrix}A_i & E_i\\ 0 & D\end{bmatrix},\;G_i=-\begin{bmatrix}C_i & Q_i\end{bmatrix},\;L_i^{xd}=\begin{bmatrix}L_i^x\\ L_i^d\end{bmatrix},$$

for $i\in\mathbb{F}$. It is clear that stability of $H_i+L_i^{xd}G_i$, $\forall i$, implies stability of $H+L^{xd}G$. Since by Assumption 5, the pairs $(G_i,H_i)$ are detectable, the corresponding $L_i^{xd}$ exist such that matrices $H_i+L_i^{xd}G_i$, $i\in\mathbb{F}$, are Hurwitz stable.

Ensuring the stability of matrix $\tilde{A}$ is a more complex task and requires some additional analysis. First, we introduce a virtual system

$$\hat{A}_i=\begin{bmatrix}A_i+B_iK_i^x & B_iK_i^{\zeta}\\ 0 & S+L_i^{\zeta}R\end{bmatrix},\hat{B}_i=\begin{bmatrix}0\\ L_i^{\zeta}\end{bmatrix},\hat{C}_i=\begin{bmatrix}C_i & 0\end{bmatrix},$$

with the corresponding transfer function

$$T_i(s)=\hat{C}_i(sI-\hat{A}_i)^{-1}\hat{B}_i.$$

Then, the following theorem gives sufficient conditions for ensuring the stability of matrix $\tilde{A}$.

**Theorem 2.** *For the matrix $\tilde{A}$ to be stable, it is sufficient that the following conditions hold:*

$$\|T_i\|_{\infty}<\gamma^*,\;i\in\mathbb{F}, \tag{18}$$

*where* $\gamma^*=\frac{1}{\rho(I-\tilde{\mathcal{L}}_1)}$.

**Proof.** Matrix $\tilde{A}$ can be rewritten as $\tilde{A}=\hat{A}+\hat{B}(\tilde{\mathcal{L}}_1-I)\hat{C}$, where

$$\hat{A}=\begin{bmatrix}A+BK^x & BK^{\zeta}\\ 0 & \tilde{S}+L^{\zeta}\tilde{R}\end{bmatrix},\;\hat{B}=\begin{bmatrix}0\\ L^{\zeta}\end{bmatrix},\;\hat{C}=\begin{bmatrix}C & 0\end{bmatrix}.$$

Assume that $\hat{A}$ is a Hurwitz stable matrix. The matrix determinant lemma gives

$$\det(sI - \tilde{A}) = \det(sI - \hat{A}) \det\left(I - (sI - \hat{A})^{-1}\hat{B}(\tilde{\mathcal{L}}_1 - I)\hat{C}\right)$$
$$= \det(sI - \hat{A}) \det\left(I + (I - \tilde{\mathcal{L}}_1)\hat{C}(sI - \hat{A})^{-1}\hat{B}\right).$$

Matrix $\tilde{A}$ is Hurwitz stable if $\det(sI - \tilde{A}) \neq 0$, $\forall s \in \bar{\mathbb{C}}^+$, which holds if

$$\sup_{s \in \bar{\mathbb{C}}^+} \rho\left((I - \tilde{\mathcal{L}}_1)\hat{C}(sI - \hat{A})^{-1}\hat{B}\right) < 1.$$

Since $\hat{C}(sI - \hat{A})^{-1}\hat{B} = \text{diag}\{T_i(s)\}$, which can be shown by rearranging the coordinates of the overall system, the equality $\rho((I - \tilde{\mathcal{L}}_1)\hat{C}(sI - \hat{A})^{-1}\hat{B}) = \rho((I - \tilde{\mathcal{L}}_1)\text{diag}\{T_i(s)\})$ follows. Furthermore, according to the block-norm matrix inequality [Lemma 7, [10]] and [Lemma 8, [10]], respectively, we get

$$\rho\left((I - \tilde{\mathcal{L}}_1)\text{diag}\{T_i(s)\}\right) \leq \rho\left(|I - \mathcal{L}_1|\text{diag}\{\|T_i\|_\infty\}\right)$$
$$\leq \rho(|I - \mathcal{L}_1|) \max_i \|T_i\|_\infty < 1.$$

Note that $\rho(|I - \mathcal{L}_1|) = \rho(I - \mathcal{L}_1) = 1/\gamma^*$. Multiplying both sides of the last inequality by $\gamma^*$ leads to the stability condition (18). □

**Remark 6.** In the case of the followers with identical dynamics, starting from the stability condition (18), it is possible to derive distinct stability condition for each agent, which depends on the specific eigenvalue of the matrix $(I - \mathcal{L}_1)$ [14]. However, in practical situations, it is often more realistic to have knowledge of only $\rho(I - \mathcal{L}_1)$, rather than all eigenvalues. In scenarios where both followers and leaders have identical dynamics, there is no need for additional distributed observers to estimate the convex estimation of the leader's states. In such cases, simpler protocols, like the one proposed in [34], can be employed. It should be noted that our result can be applied to systems that are input-output linearizable, such as Euler-Lagrange systems and nonholonomic mobile robots [4,7,12,20,34,37].

### 3.3. *Design procedure*

In this subsection, we propose a method for determining the controller gains such that matrix $A_c$ is Hurwitz stable.

It is a well known fact from $\mathcal{H}_\infty$ control theory that in general the condition (18) cannot be satisfied for an arbitrarily small $\gamma_i < \gamma^*$, where $\gamma_i > 0$ is a desired upper bound for $\|T_i\|_\infty$. Therefore, prior to discussing the design procedure, we introduce Lemma 6 in which we establish a lower bound for $\|T_i\|_\infty$. Any desired $\gamma_i$ that is smaller than the lower bound would result in a problem that does not have a solution.

**Lemma 6.** *Suppose that transfer function $T_i(s)$ is stable. Then $\|T_i\|_\infty$ is finite and*

$$\|T_i\|_\infty \geq 1, \; i \in \mathbb{F}. \tag{19}$$

**Proof.** Introduce the similarity transformation $\breve{A}_i = M_i\hat{A}_iM_i^{-1}$, $\breve{B}_i = M_i\hat{B}_i$, $\breve{C}_i = \hat{C}_iM_i^{-1}$, where

$$M_i = \begin{bmatrix} I & -\Pi_i^\zeta \\ 0 & I \end{bmatrix}.$$

This leads to the system in a state-space form, given by

$$\breve{A}_i = \begin{bmatrix} A_i + B_iK_i^x & -\Pi_i^\zeta L_i^\zeta R \\ 0 & S + L_i^\zeta R \end{bmatrix},$$
$$\breve{B}_i = \begin{bmatrix} -\Pi_i^\zeta L_i^\zeta \\ L_i^\zeta \end{bmatrix}, \; \breve{C}_i = \begin{bmatrix} C_i & R \end{bmatrix}. \tag{20}$$

Suppose that $\|T_i\|_\infty < 1$. Then, according to the small-gain theorem, the unity negative output feedback stabilizes the system (20). The resulting state matrix is then

$$\breve{A}_i - \breve{B}_i\breve{C}_i = \begin{bmatrix} A_i + B_iK_i^x + \Pi_i^\zeta L_i^\zeta C_i & 0 \\ -L_i^\zeta C_i & S \end{bmatrix},$$

which is unstable. Hence, (19) must hold. □

The structure of matrix $\hat{A}_i$ (and $\breve{A}_i$) allows the separation principle to be applied. Since it is challenging to determine $K_i^x$ and $L_i^\zeta$ simultaneously such that the condition (18) is satisfied, this will be done in two steps. In the first step, $L_i^\zeta$ is determined such that $S + L_i^\zeta R$ is Hurwitz stable, while in the second step $K_i^x$ is found such that (18) holds.

**Remark 7.** Assumption 5 guarantees the existence of $L_i^\zeta$ such that $S + L_i^\zeta R$ is Hurwitz. Standard pole placement techniques can then be utilized for finding an appropriate $L_i^\zeta$. However, an arbitrary pole placement may cause difficulties afterwards in ensuring stability of $\tilde{A}$, since the value of $\|T_i\|_\infty$ depends on the choice of $L_i^\zeta$. Therefore, in the following we introduce an ARE method for determining $L_i^\zeta$, which subsequently provides more flexibility for finding the remaining gains such that (18) holds.

The transfer function $T_i(s) = \hat{C}_i(sI - \hat{A}_i)^{-1}\hat{B}_i$ can be rewritten as

$$T_i(s) = -C_i\left(sI - (A_i + B_iK_i^x)\right)^{-1}\Pi_i^\zeta L_i^\zeta$$
$$+ \left(I - C_i\left(sI - (A_i + B_iK_i^x)\right)^{-1}\Pi_i^\zeta L_i^\zeta\right)\tilde{T}_i(s), \tag{21}$$

where $\tilde{T}_i(s) = R(sI - (S + L_i^\zeta R))^{-1}L_i^\zeta$. We present the following two lemmas concerning $\tilde{T}_i(s)$ and consequently $T_i(s)$.

**Lemma 7.** *Consider a system $\tilde{T}_i(s)$, which admits the state-space realization*

$$\dot{\xi}_i = (S + L_i^\zeta R)\xi_i + L_i^\zeta \upsilon_i$$
$$\eta_i = R\xi_i, \tag{22}$$

*and suppose that Assumptions 2 and 5 hold. Then, for any $\tilde{\gamma}_i > 1$ there exists a gain $L_i^\zeta$ such that $\|\tilde{T}_i\|_\infty < \tilde{\gamma}_i$. The gain matrix $L_i^\zeta$ can be obtained as $L_i^\zeta = -\tilde{P}_iR^T$, where $\tilde{P}_i > 0$ is a solution of the following ARE*

$$\tilde{P}_iS^T + S\tilde{P}_i + \left(\tilde{\gamma}_i^{-2} - 1\right)\tilde{P}_iR^TR\tilde{P}_i + \tilde{\epsilon}_iI = 0, \tag{23}$$

*where $\tilde{\epsilon}_i > 0$.*

**Proof.** According to the bounded real lemma [34], $S + L_i^\zeta R$ is Hurwitz stable and $\|\tilde{T}_i\|_\infty < \tilde{\gamma}_i$, if there exists a $\tilde{P}_i > 0$ such that

$$\tilde{P}_i(S + L_i^\zeta R)^T + (S + L_i^\zeta R)\tilde{P}_i + \tilde{\gamma}_i^{-2}L_i^\zeta L_i^{\zeta T} + \tilde{P}_iR^TR\tilde{P}_i < 0.$$

Let $L_i^\zeta = -\tilde{P}_iR^T$, which reduces the previous inequality to

$$\tilde{P}_iS^T + S\tilde{P}_i + (\tilde{\gamma}_i^{-2} - 1)\tilde{P}_iR^TR\tilde{P}_i < 0. \tag{24}$$

Since $S$ has eigenvalues with non negative real parts, there does not exist $\tilde{P}_i > 0$ that satisfies the inequality (24) whenever $\tilde{\gamma}_i^{-2} - 1 \geq 0$, thus $\tilde{\gamma}_i > 1$ must hold.

The inequality (24) is satisfied for a $\tilde{P}_i > 0$ that is a solution of the ARE (23). The existence of such $\tilde{P}_i$ is ensured by the detectability of $(R, S)$ [13]. □

Note that $L_i^\zeta = -\tilde{P}_iR^T$ is not a unique substitution for the gain $L_i^\zeta$ in terms of $\tilde{P}_i$. However, the choice leads to the minimum possible norm of $\tilde{T}_i(s)$. The value of the norm is established in the following.

**Lemma 8.** *For a system $\tilde{T}_i(s)$ with a state-space realization given by (22), the inequality $\|\tilde{T}_i\|_\infty \geq 1$ holds.*

**Proof.** Under the feedback controller $\upsilon_i = -\eta_i$, the closed-loop matrix of (22) is equal to $S$, which is an anti-Hurwitz stable as stated in Assumption 2. Therefore, according to the small-gain theorem, $\|\tilde{T}_i\|_\infty \geq 1$ must hold. □

**Remark 8.** Lemmas 7 and 8 imply that there always exists a gain matrix $L_i^\zeta$ such that $\|\tilde{T}_i\|_\infty \in [1, \tilde{\gamma}_i)$, where $\tilde{\gamma}_i > 1$ can be chosen arbitrarily close to 1.

**Remark 9.** Suppose that $\tilde{\gamma}_i$ is chosen as $\tilde{\gamma}_i = 1 + \Delta_i$, where $\Delta_i > 0$ is a small constant. Then, $\|\tilde{T}_i\|\|_\infty \to 1^+$ is guaranteed. Furthermore, it is a well known fact that the solution $\tilde{P}_i$ of the ARE (23) is monotone non-decreasing with respect to $\tilde{\epsilon}_i$ [35]. Therefore, by decreasing $\tilde{\epsilon}_i$, $L_i^\zeta$ will also decrease, leading to a smaller $\|T_i\|_\infty$, as it can be seen from (21), thus giving more flexibility for choosing $K_i^x$ afterwards.

The next step is to find an appropriate $K_i^x$ such that $T_i(s)$ is stable and that the condition (18) is satisfied. In order to approach this problem, rewrite $\hat{A}_i$ as $\hat{A}_i = \bar{A}_i + \bar{B}_i K_i^x \bar{C}_i$, where

$$\bar{A}_i = \begin{bmatrix} A_i & B_i\Gamma_i^\zeta \\ 0 & S + L_i^\zeta R \end{bmatrix},\ \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix},\ \bar{C}_i = \begin{bmatrix} I & -\Pi_i^\zeta \end{bmatrix}.$$

The transfer function is then equal to $T_i(s) = \hat{C}_i(sI - \bar{A}_i - \bar{B}_i K_i^x \bar{C}_i)^{-1}\hat{B}_i$. The task of finding $K_i^x$ such that (18) holds can be viewed as an $\mathcal{H}_\infty$ Static Output Feedback (SOF) problem, and there are many approaches in the literature for tackling this problem. We will take the advantage of the iterative LMI (ILMI) approach [9], denoted here as Algorithm 1, which we present for the sake of completeness.

Note that due to Assumption 4, structure of the matrix $\hat{A}_i$ and Hurwitzness of $S + L_i^\zeta R$, the triplet $(\bar{A}_i, \bar{B}_i, \bar{C}_i)$ is output feedback stabilizable. That is, there exists $K_i^x$ such that $\bar{A}_i + \bar{B}_i K_i^x \bar{C}_i$ is Hurwitz stable. From this follows that the pair $(\bar{A}_i, \bar{B}_i)$ is stabilizable, since the gain $K_i^x \bar{C}_i$ stabilizes it, and similarly $(\bar{C}_i, \bar{A}_i)$ is detectable, with the stabilizing gain $\bar{B}_i K_i^x$. This is important since Algorithm 1 requires stabilizability of the pair $(\bar{A}_i, \bar{B}_i)$ and detectability of $(\bar{C}_i, \bar{A}_i)$. After finding appropriate gain $K_i^x$, remaining gains $K_i^d$ and $K_i^\zeta$ can be obtained from (13).

Final step in the design procedure is to find $L_i^{xd}$ such that $H_i + L_i^{xd} G_i$, $i \in \mathbb{F}$, are Hurwitz. For this, we will use the ARE method described in [17]. The gain $L_i^{xd}$ is found as $L_i^{xd} = -P_i G_i^T$, where $P_i > 0$ is a unique solution of ARE

$$H_i P_i + P_i H_i^T - P_i G_i^T G_i P_i + \epsilon_i I = 0, \tag{25}$$

for an $\epsilon_i > 0$. Calculating the gain $L_i^{xd}$ for every follower in this way guarantees the stability of $H + L^{xd} G$.

Finally, for the sake of clarity, we concisely present the design procedure for determining the controller gains in Algorithm 2, which should be repeated for each follower.

**Algorithm 1** $\mathcal{H}_\infty$ SOF procedure [9].

1: Set $\mathcal{P}_0 = I$ and $\mathcal{Q}_0 = I$.

2: Solve the following optimization problem for $\mathcal{P}, \mathcal{Q}, \mathcal{V}_1, \mathcal{V}_2$

$$\min\quad \mathrm{trace}(\mathcal{P}\mathcal{Q}_0 + \mathcal{Q}\mathcal{P}_0),\ \text{s.t. constraints}$$
$$\begin{pmatrix} \mathcal{P}\bar{A}_i + \bar{A}_i^T\mathcal{P} + \mathcal{V}_1\bar{C}_i + \bar{C}_i^T\mathcal{V}_1^T & \mathcal{P}\hat{B}_i & \hat{C}_i^T \\ \hat{B}_i^T\mathcal{P} & -\gamma_i I & 0 \\ \hat{C}_i & 0 & -\gamma_i I \end{pmatrix} < 0,$$
$$\begin{pmatrix} \bar{A}_i\mathcal{Q} + \mathcal{Q}\bar{A}_i^T + \bar{B}_i\mathcal{V}_2 + \mathcal{V}_2^T\bar{B}_i^T & \hat{B}_i & \mathcal{Q}\hat{C}_i^T \\ \hat{B}_i^T & -\gamma_i I & 0 \\ \hat{C}_i\mathcal{Q} & 0 & -\gamma_i I \end{pmatrix} < 0,$$
$$\begin{pmatrix} \mathcal{P} & I \\ I & \mathcal{Q} \end{pmatrix} \geq 0.$$

3: Check the following conditions:

1. if $\mathrm{trace}(\mathcal{P}\mathcal{Q}) - n_i < \delta_1$, a prescribed tolerance, go to Step 4
2. if $\mathrm{trace}(\mathcal{P}\mathcal{Q}) - \mathrm{trace}(\mathcal{P}_0\mathcal{Q}_0) < \delta_2$, a prescribed tolerance, a solution may not exist, EXIT
3. otherwise, set $\mathcal{P}_0 = \mathcal{P}$, $\mathcal{Q}_0 = \mathcal{Q}$ and go to Step 2.

4: Set $\mathcal{P}_0 = \mathcal{P}$. Solve the following optimization problem for $K_i^x$ with given $\mathcal{P}$

$$\min\quad \alpha,\ \text{s.t.constraint}$$
$$\begin{pmatrix} \Phi & \mathcal{P}\hat{B}_i & \hat{C}_i^T \\ \hat{B}_i^T\mathcal{P} & -\gamma_i I & 0 \\ \hat{C}_i & 0 & -\gamma_i I \end{pmatrix} < 0,$$

where $\Phi = \mathcal{P}\bar{A}_i + \bar{A}_i^T\mathcal{P} + \mathcal{P}\bar{B}_i K_i^x \bar{C}_i + \bar{C}_i^T K_i^{xT} \bar{B}_i^T \mathcal{P} - \alpha\mathcal{P}$.

5: if $\alpha \leq 0$, the stabilizing gain $K_i^x$ is found, EXIT.

6: Solve the following optimization problem for $\mathcal{P}$ with given $K_i^x$:

min $\alpha$, s.t constraint from Step 4.

7: if $\alpha < 0$, the stabilizing gain $K_i^x$ is found, EXIT.

8: Solve the following optimization problem for $\mathcal{P}$ with given $K_i^x$ and $\alpha$:

$\min \mathrm{trace}(\mathcal{P})$, s.t. constraint from Step 4.

9: Check the following conditions:

1. if $\|\mathcal{P} - \mathcal{P}_0\|/\|\mathcal{P}\| < \delta$, a prescribed tolerance, the solution may not exist, EXIT
2. otherwise go to Step 4

**Algorithm 2** Controller Design Algorithm.

1: Initialize: $1 < \gamma_i \leq \gamma^*$, $\tilde{\gamma}_i > 1$, $\epsilon_i > 0$ and $\tilde{\epsilon}_i > 0$.

2: Solve

$$S\tilde{P}_i + \tilde{P}_i S^T + (\tilde{\gamma}_i^{-2} - 1)\tilde{P}_i R^T R \tilde{P}_i + \tilde{\epsilon}_i I = 0,$$

for $\tilde{P}_i > 0$. Compute $L_i^\zeta = -\tilde{P}_i R^T$.

3: Compute $K_i^x$ by Subalgorithm 1 for $\gamma_i < \gamma^*$. Then, $K_i^\zeta = \Gamma_i^\zeta - K_i^x \Pi_i^\zeta$ and $K_i^d = \Gamma_i^d - K_i^x \Pi_i^d$, where $(\Pi_i^\zeta, \Gamma_i^\zeta)$, $(\Pi_i^d, \Gamma_i^d)$ are the solutions to the regulator equations (4).

4: If Subalgorithm 1 does not return the stabilizing solution, return to Step 1, decrease $\tilde{\epsilon}_i$ and increase $\gamma_i$. If Subalgorithm 1 returns the stabilizing solution, but performance is not satisfactory, return to Step 1 and increase $\tilde{\epsilon}_i$. Otherwise, go to Step 5.

5: Solve

$$H_i P_i + P_i H_i^T - P_i G_i^T G_i P_i + \epsilon_i I = 0,$$

for a unique $P_i > 0$. Compute $L_i^{xd} = -P_i G_i^T$.

## 4. Solvability analysis

This section is concerned with establishing theoretical guarantees for an existence of a solution in the form of the controller (8) to the output containment control problem of MAS (1)–(3). The analysis takes place from three different viewpoints: leaders' dynamics, followers' dynamics and graph topology.

### 4.1. Leaders' dynamics

In the following, it will be shown that a solution to the output containment problem always exists when we substitute Assumption 2 with Assumption 7.

**Assumption 7.** The eigenvalues of $S$ lie on the imaginary axis, i.e. $\mathrm{Re}(\lambda(S)) = 0$.

**Corollary 1.** *Suppose that Assumptions 1, 3–7 hold. Then, the output containment control objective can always be achieved by MAS (1)–(3) under the proposed protocol (8).*

**Proof.** As it was already stated in Remark 9, a solution $\tilde{P}_i$ of (23) is monotone non-decreasing with respect to $\tilde{\epsilon}_i$. In a special case, when matrix $S$ has eigenvalues in $\bar{\mathbb{C}}^-$, $\tilde{\epsilon}_i \to 0$ corresponds to $\tilde{P}_i \to 0$ [17], thus leading to $L_i^\zeta \to 0$. This encompasses the case when Assumption 7 holds. Therefore, for any gain $K_i^x$ that stabilizes

$A_i + B_i K_i^x$, it follows that $T_i(s) \to \tilde{T}_i(s)$, as it can clearly be seen from (21). The choice of $\tilde{\gamma}_i \to 1^+$ in (23) leads to $\|\tilde{T}_i\|_\infty \to 1^+$, thus implying $\|T_i\|_\infty \to 1^+$. By the means of Lemma 2 and the definition (18) of $\gamma^*$, it is straightforward to conclude that $\gamma^* > 1$. □

**Remark 10.** It should be noted that larger $\bar{\epsilon}_i$ leads to the increase of $\tilde{P}_i$ and consequently $L_i^\zeta$, resulting in a system with a faster response. Although in this case it also holds $\|\tilde{T}_i\|_\infty \to 1^+$, the other terms in $T_i(s)$ become non-negligible, thus leading to a larger $\|T_i\|_\infty$. Therefore, one should initialize Algorithm 1 with a larger $\gamma_i < \gamma^*$ and gradually decrease $\gamma_i$ until a solution is found, as described in Algorithm 2.

### 4.2. Followers' dynamics

Let the following assumption hold, which means that the followers' dynamics is minimum-phase and right-invertible.

**Assumption 8.** Matrices $(A_i, B_i, C_i)$, $i \in \mathbb{F}$, satisfy

$$\text{rank}\left(\begin{bmatrix} A_i - sI & B_i \\ C_i & 0 \end{bmatrix}\right) = n_i + p, \ \forall s \in \bar{\mathbb{C}}^+.$$

**Remark 11.** The right-invertibility of the $i$th agent's dynamics basically means that there exists an initial condition $x_i(0)$ and an input $u_i(t)$ such that $y_i(t)$ is equal to the reference output, for all $t \geq 0$. An example of right-invertible system is single-input single-output (SISO) system whose transfer function is not identically zero [6]. Furthermore, it should be noted that Assumption 6 is encompassed within Assumption 8.

Let $F_i(s) = C_i(sI - (A_i + B_i K_i^x))^{-1} \Pi_i^\zeta L_i^\zeta$. The transfer function in (21) can be rewritten as $T_i(s) = -F_i(s)(I + \tilde{T}_i(s)) + \tilde{T}_i(s)$. Then, we have the following corollary.

**Corollary 2.** *Let Assumptions 1–5 and 8 hold and consider MAS (1)–(3) and controller (8). Then, the output containment problem is always solvable.*

**Proof.** The right-invertibility of $(A_i, B_i, C_i)$ implies the left-invertibility of the system $(A_i^T, C_i^T, B_i^T)$. According to [Lemma 3.1, [27]], under Assumptions 4 and 8, $K_i^x$ can be found such that $\|F_i^T\|_\infty \leq f_i$ for any $f_i > 0$, where $F_i^T(s) = (\Pi_i^\zeta L_i^\zeta)^T (sI - (A_i + B_i K_i^x)^T)^{-1} C_i^T$. Since $\|F_i^T\|_\infty = \|F_i\|_\infty$, this implies that $\|F_i\|_\infty$ can be made arbitrarily small. By making use of standard norm properties we get $\|T_i\|_\infty \leq \|F_i\|_\infty \|I + \tilde{T}_i\|_\infty + \|\tilde{T}_i\|_\infty$. Therefore, $\|F_i\|_\infty \|I + \tilde{T}_i\|_\infty$ can be made arbitrarily small, which leaves the condition $\|\tilde{T}_i\|_\infty < \gamma^*$. This can always be satisfied for any $1 < \tilde{\gamma}_i < \gamma^*$, as stated in Remark 8. □

**Remark 12.** In Corollary 2, it is shown that a solution to the output containment control problem always exists for the followers with right-invertible and minimum-phase dynamics, regardless of the leaders' dynamics. On the other hand, Corollary 1 claims that a solution to the output containment problem always exists for the leaders with poles on the imaginary axis, regardless of the followers' dynamics. It should be noted that even though in the most general case, when $\text{Re}(\lambda(S)) \geq 0$, the existence of a solution cannot be guaranteed, this does not mean that a solution does not exist. Moreover, the most practically interesting scenario arises when matrix $S$ satisfies Assumption 7. In this case, the exosystem can generate a diverse range of reference signals, including step signals, polynomial signals, sinusoidal signal of various frequency, and their linear combinations [25].

### 4.3. Acyclic graphs

In this subsection, it is shown that for acyclic graphs it is sufficient to ensure the stability of $A_i + B_i K_i^x$, $S + L_i^\zeta R$ and $H_i + L_i^{xd} G_i$, $\forall i \in \mathbb{F}$ in order to solve the output containment problem.

**Assumption 9.** The digraph $\mathcal{G}$ is acyclic and contains a united spanning tree.

**Corollary 3.** *Consider the MAS (1)–(3) and the controller (8). Then, under Assumptions 2–6 and 9, the output containment objective can always be achieved.*

**Proof.** Since the digraph is acyclic, this means that the nodes can be rearranged such that the Laplacian matrix $\mathcal{L}$ is lower triangular. Furthermore, this implies that $\mathcal{L}_1$ is also lower triangular, and since $\lambda(I - \mathcal{L}_1) = 1 - \lambda(\mathcal{L}_1)$ it can be concluded that all eigenvalues of matrix $I - \mathcal{L}_1$ are zero. Hence, $\gamma^*$ in (18) becomes infinite. Therefore, it becomes sufficient to find the controller gains such that $\|T_i\|_\infty$ is finite, which is the same as ensuring stability of $A_i + B_i K_i^x$ and $S + L_i^\zeta R$. This is always possible under Assumptions 4 and 5, respectively. □

## 5. Simulation results

In this section, we provide three numerical examples to demonstrate the effectiveness of the proposed approach.

*Example 1: General linear models*

Consider a multi-agent system consisting of four followers $\mathbb{F} = \{1, 2, 3, 4\}$ and three leaders $\mathbb{L} = \{5, 6, 7\}$, with the interaction network represented by the digraph $\mathcal{G}$, which is defined by its Laplacian matrix

$$\mathcal{L} = \frac{1}{3}\begin{bmatrix} 3 & -1.5 & 0 & 0 & -1.5 & 0 & 0 \\ 0 & 3 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & 3 & 0 & 0 & -1 & -1 \\ -1 & 0 & -1 & 3 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (26)$$

The dynamics of the leaders is given by

$$S = \begin{bmatrix} 1 & -3 \\ 1 & -1 \end{bmatrix}, \ R = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad (27)$$

while the exosystem generating disturbances is described by

$$D = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}. \quad (28)$$

The dynamics of the followers is given in the state-space form

$$\begin{cases} A_i = \begin{bmatrix} 0.3 & -2 \\ 0.1 & -0.2 \end{bmatrix}, \ B_i = \begin{bmatrix} 1.8 \\ 0.9 \end{bmatrix} \\ C_i = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ i = \{1, 3\} \end{cases}$$

$$\begin{cases} A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.25 & 0.25 & 1 \end{bmatrix}, \ B_i = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix} \\ C_i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \ i = \{2, 4\} \end{cases}$$

and

$$E_1 = \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}, \ E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \ E_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$E_4 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix},\ Q_i = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ i = \{1, 2, 3, 4\}.$$

Solutions of the first pair of the regulator Eq. (4) are

$$\Pi_i^{\zeta} = \begin{bmatrix} 1.00 & 1.00 \\ 0.13 & 0.92 \end{bmatrix},\ \Gamma_i^{\zeta} = \begin{bmatrix} 1.09 & -1.36 \end{bmatrix},\ \text{for } i = \{1, 3\},$$

$$\Pi_i^{\zeta} = \begin{bmatrix} 1.00 & 1.00 \\ 0.17 & 0.57 \\ -0.17 & 1.21 \end{bmatrix},\ \Gamma_i^{\zeta} = \begin{bmatrix} 1.83 & -4.57 \end{bmatrix},\ \text{for } i = \{2, 4\},$$

while the second pair of regulator equations admits solutions

$$\Pi_1^{d} = \begin{bmatrix} -1.00 & 1.00 \\ 0.19 & 1.15 \end{bmatrix},\ \Pi_2^{d} = \begin{bmatrix} -1.00 & 1.00 \\ -0.30 & 0.48 \\ 0.39 & 0.64 \end{bmatrix},$$

$$\Pi_3^{d} = \begin{bmatrix} -1.00 & 1.00 \\ 0.23 & 0.63 \end{bmatrix},\ \Pi_4^{d} = \begin{bmatrix} -1.00 & 1.00 \\ -0.13 & 0.21 \\ -0.48 & 0.35 \end{bmatrix},$$

$$\Gamma_1^{d} = \begin{bmatrix} -1.29 & 0.56 \end{bmatrix},\ \Gamma_2^{d} = \begin{bmatrix} -2.70 & -2.48 \end{bmatrix},$$

$$\Gamma_3^{d} = \begin{bmatrix} -1.25 & -0.57 \end{bmatrix},\ \Gamma_4^{d} = \begin{bmatrix} -1.87 & -3.21 \end{bmatrix}.$$

Furthermore, we set $\gamma_i = 1.5$, $\epsilon_i = 10^2$, $\bar{\epsilon}_i = 10^3$ and $\tilde{\gamma}_i = 1.41$ for all $i \in \mathbb{F}$. Note that $\gamma_i$ satisfies the condition $\gamma_i < \gamma^*$, since $\gamma^* = 2.21$. Following the design procedure presented in Algorithm 2 leads to the observer gains

$$L_1^x = \begin{bmatrix} 10.68 \\ -7.19 \end{bmatrix},\ L_2^x = \begin{bmatrix} 35.63 \\ 76.91 \\ 106.21 \end{bmatrix},\ L_3^x = \begin{bmatrix} 22.16 \\ -3.30 \end{bmatrix},\ L_4^x = \begin{bmatrix} 40.12 \\ 76.28 \\ 105.35 \end{bmatrix},$$

$$L_1^d = \begin{bmatrix} 13.64 \\ 3.72 \end{bmatrix},\ L_2^d = \begin{bmatrix} 1.20 \\ 14.09 \end{bmatrix},\ L_3^d = \begin{bmatrix} 9.19 \\ 10.75 \end{bmatrix},\ L_4^d = \begin{bmatrix} -3.65 \\ 13.66 \end{bmatrix},$$

$$L_i^{\zeta} = \begin{bmatrix} -78.79 \\ 9.62 \end{bmatrix},\ i \in \mathbb{F},$$

and control law gains

$$K_i^x = \begin{bmatrix} 1.18 & -4.75 \end{bmatrix},\ i = \{1, 3\},$$
$$K_i^x = \begin{bmatrix} -0.27 & 0.33 & -6.71 \end{bmatrix},\ i = \{2, 4\}.$$

This results in $\|T_1\|_\infty = \|T_3\|_\infty = 1.2121$, $\|T_2\|_\infty = \|T_4\|_\infty = 1.2114$, which means that the stability condition (18) is satisfied. Furthermore, since the matrix $H + L^{xd}G$ is Hurwitz stable, Lemma 5 implies that the output containment control objective will be achieved.

Initial states of the agents are: $x_1(0) = [1 \quad 0]^T$, $x_2(0) = [1 \quad 0 \quad 0]^T$, $x_3(0) = [1 \quad 0]^T$, $x_4(0) = [1 \quad 0 \quad 0]^T$, $\zeta_5(0) = [3 \quad 1.5]^T$, $\zeta_6(0) = [-0.5 \quad -1.5]^T$, $\zeta_7(0) = [-2 \quad 1.5]^T$, while the initial state of the disturbance generating exogenous system is $d(0) = [0 \quad 1]^T$. The initial conditions of all observers of each follower are set to zero.

From Fig. 1 it is evident that the local neighborhood output containment error converges to zero. The output trajectories of the followers are shown in Fig. 2. As can be observed, the followers' outputs converge and stay inside the convex hull generated by the leaders' outputs. The state trajectories of agents and observers are shown in Figs. 3, 4 and 5. It can be seen that the states of the local observers converge to true values within 5 s (Figs. 3 and 4). On the other hand, the state trajectories of the distributed observer converge to the convex combination of the leaders' states (Fig. 5).

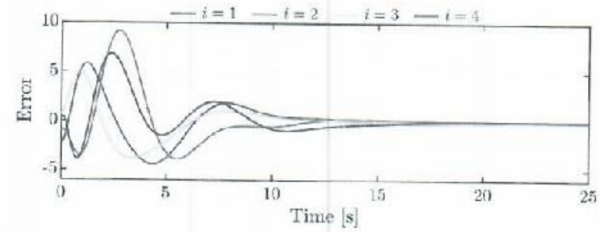



**Fig. 1.** The local output containment error \unboldmath $e_i$, $i \in \mathbb{F}$.

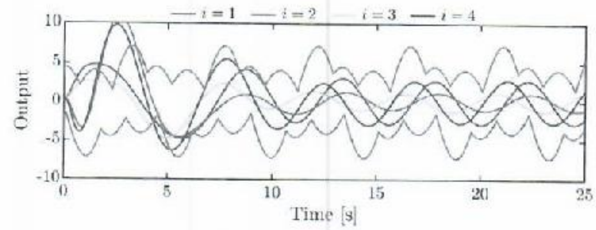



**Fig. 2.** The followers' outputs and convex hull generated by leaders' outputs.

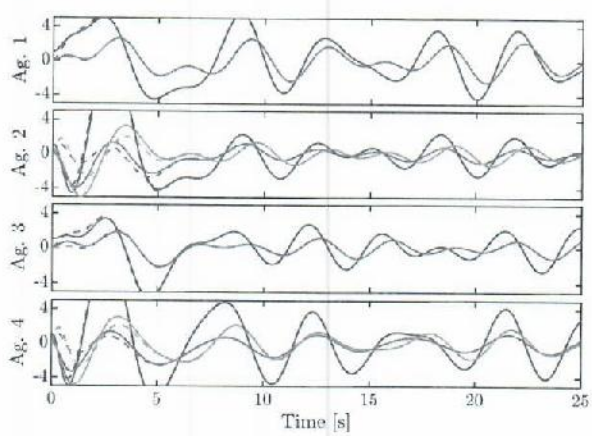



**Fig. 3.** States of the agents (solid lines) and observers (dashed lines).

*Example 2: Caltech wireless testbed vehicles*

In this example, we consider a multi-agent system consisting of five followers $\mathbb{F} = \{1, 2, 3, 4, 5\}$ and three leaders $\mathbb{L} = \{6, 7, 8\}$, with the interaction network represented by the digraph $\mathcal{G}$, which is defined by its Laplacian matrix

$$\mathcal{L} = \frac{1}{2} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 0 & -1 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 & -2 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (29)$$

The exosystem matrices are the same as in the previous example. The only difference is that in this example, the leaders have two outputs defined by matrix $R = I_2$. Note that the eigenvalues of $S$ are $\pm j\sqrt{2}$, which means that the leaders' trajectories will be ellipses in the 2D plane.

The followers in this example are Caltech testbed vehicles, described by sixth order linearized models [12,34,38]. As in [38], the vehicles differ in mass, with $m_i = 0.7$, $i \in \{1, 3, 5\}$ and $m_i = 0.8$, $i \in \{2, 4\}$. All other parameters are consistent with those in [12]. The controlled outputs are the two first states that represent

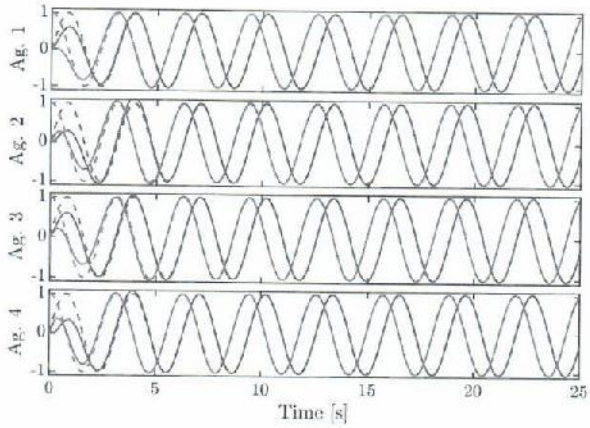



**Fig. 4.** States of the local disturbance observer for each agent (solid lines) and disturbance generating exosystem (dashed lines).

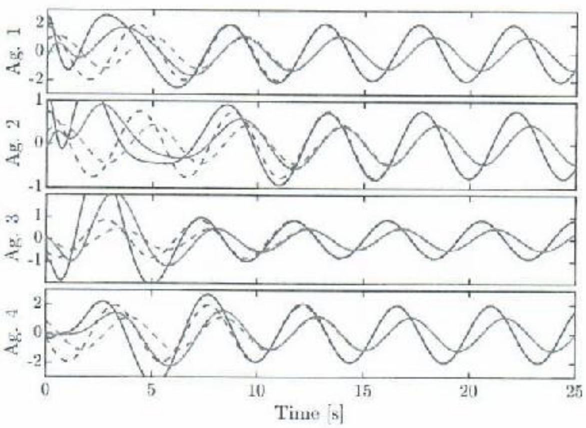



**Fig. 5.** Distributed observer states (solid lines) and convex combinations of leaders' states to which they converge (dashed lines).

the positions along the $x$ and $y$ coordinates, i.e. $C_i = \begin{bmatrix} I_2 & 0_{2\times 4} \end{bmatrix}$. The impact of disturbances on the vehicles is defined by matrices $E_i = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$, $Q_i = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\forall i \in \mathbb{F}$.

Solutions of the first pair of the regulator Eq. (4) are

$$\Pi_i^\zeta = \begin{bmatrix} 1 & 0 & 4.67 & 1 & 1 & 1 \\ 0 & 1 & -3.67 & -3 & -1 & -10.33 \end{bmatrix}^T,$$

$$\Gamma_i^\zeta = \begin{bmatrix} -0.25 & -0.55 \\ -0.52 & -0.87 \end{bmatrix}, \; i \in \{1, 3, 5\},$$

$$\Pi_i^\zeta = \begin{bmatrix} 1 & 0 & 5.33 & 1 & 1 & 1 \\ 0 & 1 & -4.33 & -3 & -1 & -11.67 \end{bmatrix}^T,$$

$$\Gamma_i^\zeta = \begin{bmatrix} -0.30 & -0.601 \\ -0.62 & -0.95 \end{bmatrix}, \; i \in \{2, 4\},$$

while the second pair of regulator equations admits solutions

$$\Pi_i^d = \begin{bmatrix} 0 & -1 & 14 & 1 & 1 & 3 \\ -1 & 0 & -1.5 & 1 & -2 & 28 \end{bmatrix}^T,$$

$$\Gamma_i^d = \begin{bmatrix} 1.49 & 0.05 \\ -0.29 & 1.83 \end{bmatrix}, \; i \in \{1, 3, 5\},$$

$$\Pi_i^d = \begin{bmatrix} 0 & -1 & 16 & 1 & 1 & 3 \\ -1 & 0 & -1.5 & 1 & -2 & 32 \end{bmatrix}^T,$$

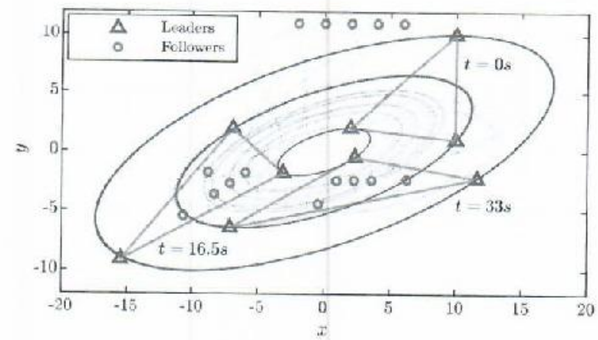



**Fig. 6.** Trajectories of the leaders and followers in a 2D plane. The leaders, denoted by triangles, move in ellipses, thus forming a time-varying triangle. The followers converge to the triangle, thus achieving the output containment objective.

$$\Gamma_i^d = \begin{bmatrix} 1.70 & 0.07 \\ -0.37 & 2.08 \end{bmatrix}, \; i \in \{2, 4\}.$$

Given that $\gamma^* = 2$ for $\mathcal{G}$, we choose the following values for the design algorithm: $\gamma_i = 1.2$, $\epsilon_i = 400$, $\bar{\epsilon}_i = 10^3$ and $\tilde{\gamma}_i = 1.0541$ for all $i \in \mathbb{F}$. By following the design procedure outlined in Algorithm 2, we obtain the following observer gains

$$L_i^x = \begin{bmatrix} 27.96 & 12.01 & -7.20 & 15.03 & 5.28 & -0.13 \\ -22.80 & 42.52 & 22.27 & -12.90 & 18.89 & 0.18 \end{bmatrix}^T, \quad i \in \{1, 3, 5\},$$

$$L_i^x = \begin{bmatrix} 28.15 & 12.12 & -7.09 & 15.29 & 5.77 & -0.12 \\ -22.78 & 42.44 & 22.31 & -12.59 & 18.77 & 0.16 \end{bmatrix}^T, \quad i \in \{2, 4\},$$

$$L_i^d = \begin{bmatrix} -12.08 & -11.66 \\ 1.22 & 22.73 \end{bmatrix}, \; i \in \{1, 3, 5\},$$

$$L_i^d = \begin{bmatrix} -12.18 & -11.59 \\ 1.05 & 22.72 \end{bmatrix}, \; i \in \{2, 4\},$$

$$L_i^\zeta = \begin{bmatrix} -112.49 & 7.72 \\ 7.72 & -89.43 \end{bmatrix}, \; i \in \mathbb{F},$$

and control law gains

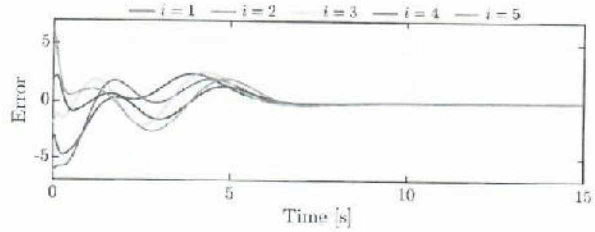
$$K_i^x = \begin{bmatrix} -59.67 & -55.12 & 0.26 & -8.17 & -5.28 & -0.03 \\ -53.63 & -57.85 & -0.63 & -4.27 & -9.18 & -0.25 \end{bmatrix}, \quad i \in \{1, 3, 5\},$$

$$K_i^x = \begin{bmatrix} -69.85 & -62.54 & 0.41 & -10.64 & -5.00 & 0.06 \\ -63.43 & -65.58 & -0.46 & -6.30 & -9.34 & -0.17 \end{bmatrix}, \quad i \in \{2, 4\},$$

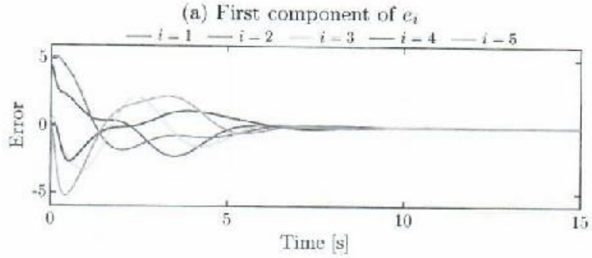
while $K_i^d$ and $K_i^\zeta$ can be easily computed using Eq. (13). The calculated parameters yield $\|T_1\|_\infty = \|T_3\|_\infty = \|T_5\|_\infty = 1.07$, $\|T_2\|_\infty = \|T_4\|_\infty = 1.06$, which means that the stability condition (18) is satisfied.

The initial state of the $i$th agent is $x_i(0) = [-5 + 2i \;\; 11 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T$, while the initial state of the disturbance generating exogenous system is $d(0) = [0 \quad 1]^T$. Additionally, the initial conditions of all observers for each follower are set to zero.

Motion of the agents in a 2D plane is shown in Fig. 6, where the triangles and circles denote the leaders and followers, respectively. It can be seen that initially the followers do not belong to the convex hull generated by the leaders' outputs. However, in steady-state, the followers converge and stay in the convex hull. Equivalently, the local neighborhood output containment error signal $e_i$, $i \in \mathbb{F}$, converges to zero, as it can be seen in Fig. 7a and b,

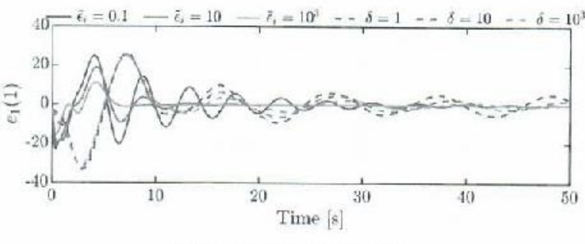



(a) First component of $e_i$

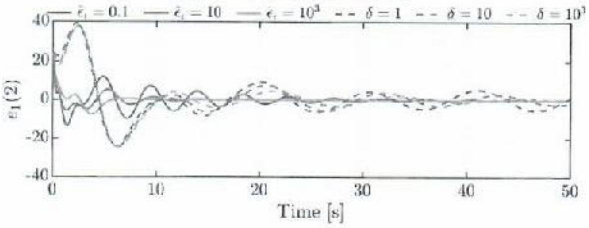



(b) Second component of $e_i$

**Fig. 7.** The local output containment error.




(a) First component of $e_1$




(b) Second component of $e_1$

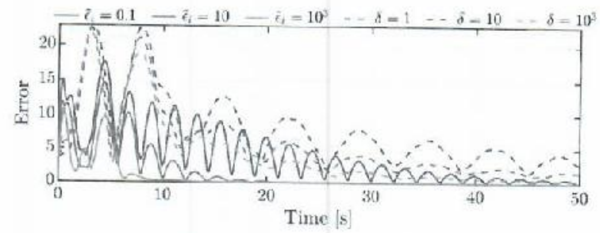
**Fig. 8.** Comparison of the containment error of the first agent when using the proposed design method (solid lines) versus the design method described in [19] (dashed lines).

thus implying that the output containment objective is achieved, as stated in Lemma 3.

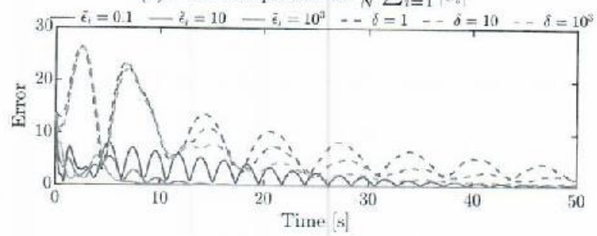
*Example 3: Comparative analysis*

In this example, we conduct further analysis of the MAS discussed in the previous example. The local observer gains remain the same, while the distributed observer gains are calculated for different values of the parameter $\bar{c}_i$. Additionally, we make comparisons with the low-gain design method presented in [19]. In this method, the controller gains are first determined to ensure the stability of the matrix $A_i + B_i K_i^x$. Specifically, we employ the standard LQR Riccati equation to calculate $K_i^x$, using $Q = \delta I$ and $R = I$, where the value of $\delta$ is varied. Subsequently, the observer gains are computed by adjusting the parameter of the Riccati equation outlined in [19]. For the sake of simplicity, the values of the observer and controller gains are not provided.

Figures 8 and 9 depict the neighborhood containment error of the first agent and the mean absolute neighborhood containment error for all agents, respectively, for various values of $\bar{c}_i$ and $\delta$.




(a) First component of $\frac{1}{N}\sum_{i=1}^{N}|e_i|$




(b) Second component of $\frac{1}{N}\sum_{i=1}^{N}|e_i|$

**Fig. 9.** Comparison of the mean absolute local containment error when using the proposed design method (solid lines) versus the design method described in [19] (dashed lines).

It can be observed that as $\bar{c}_i$ increases, the system reaches the steady-state faster. This result is expected since $L_i^\zeta$ increases with higher values of $\bar{c}_i$. In the case of using the second method, increasing $\delta$ leads to a somewhat faster response. However, it is worth noting that the errors still exhibit slow convergence and large oscillations compared to the proposed method. The primary reason for these performance differences lies in the fact that in the proposed method, the gain $L_i^\zeta$ is determined first, followed by the calculation of $K_i^x$ using the SOF algorithm. In contrast, in the low-gain method, the stabilizing gain $K_i^x$ can be chosen arbitrarily, and subsequently, the gain $L_i^\zeta$ is determined by adjusting the parameters of the Riccati equation until stability is achieved.

## 6. Conclusion

This paper deals with the output containment control problem in linear heterogeneous multi-agent systems subject to external disturbances. The main contribution of the paper is a novel distributed observer-based protocol which ensures the achievement of the output containment objective, without the exchange of the internal controller states. A design procedure is proposed and solvability analysis is carried out in detail. Theoretical guarantees for an existence of a solution are established.

Future work will be focused on extending the proposed protocol for solving the bipartite output containment problem, as well as achieving the time-varying formation output containment control. Furthermore, a special focus will be on analyzing MASs where the followers have unknown nonlinearities in their dynamics, while the leaders remain linear.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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THEORY

# Distributed Observer Approach to Cooperative Output Regulation of Multi-Agent Systems Without Exchange of Controller States

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**ABSTRACT** In this paper, the cooperative output regulation (COR) of heterogeneous linear multi-agent systems is studied. We propose a novel distributed observer approach that ensures synchronization of agents' outputs to a reference trajectory generated by a leader, while rejecting disturbance. A unified framework based on tools from $\mathcal{H}_\infty$ theory is established which allows treatment of both networks of non-introspective and networks of introspective agents. The proposed protocols do not require exchange of the controller states, which reduces or completely eliminates communication costs. A sufficient local stability condition is derived and a novel controller gain design method is provided to satisfy that condition. It is proven that the solvability of the COR problem can be guaranteed in advance for: i) introspective agents with arbitrary dynamics, ii) non-introspective agents with stable dynamics, under the assumption that the poles of exosystems lie on the imaginary axis. The effectiveness of the proposed approach is verified through numerical simulations.

**INDEX TERMS** Cooperative output regulation, leader-following consensus, observer-based approach, multi-agent systems (MASs), $\mathcal{H}_\infty$ static output feedback.

## I. INTRODUCTION

During the past two decades, cooperative control of multi-agent systems (MASs) has received significant attention from researchers, see [1], [2], [3], [4], [5] and references therein. It has been demonstrated that in order to achieve collective goals, dynamic agents need to mutually interact, which leads to new theoretical challenges that require extension of the classical methods in control. In the process, the cooperative output regulation (COR) has stood out as an important problem, where the main challenge is to design a distributed control law such that agents asymptotically track the reference trajectory generated by a leader while simultaneously rejecting disturbances. A variety of cooperative control problems, such as leaderless synchronization, leader-following consensus, formation and containment control [5], [6], [7], [8], [9], [10], [11], [12] can be seen as a special case or extension of the COR problem.

There are two well-known methods for tackling the COR problem for heterogeneous MASs. The first method is based on a distributed observer and relies on the assumption that a solution of the corresponding regulator equations exists. The pioneering work in this area was done in [13], where the dynamic compensator in form of a distributed observer was introduced. This work was extended in [14] to the output feedback case. The second method is based on the distributed internal model, which is more robust against plant parameters variation, but requires the transmission-zero condition to be satisfied [15]. Both design methods have been extensively investigated in recent years. For instance, in [16], the authors have analyzed global output regulation problem under the communication constraint of limited bandwidth. The COR problem under switching graphs and time-delays is considered in [17], [18], and [19], while adaptive protocols that

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solve the COR problem in the case when the model of a leader is not available to all agents can be found in [20] and [21].

All results in the area of cooperative control can be categorized in regard to information available to every agent in the network. More specifically, if agents possess some type of self-knowledge, such as measurement of their own state or output, then we refer to them as *introspective agents*. In this case, the possibilities of manipulating agents' internal dynamics are broad and various control schemes can be utilized. Design of protocols based on the full state information has been carried out in [13], [15], [18], [19], [20], and [21], while the protocols requiring only output information were addressed in [14], [16], and [22]. Unlike the introspective agents, if the agents in the network cannot measure their own state or output, then they are called *non-introspective agents* [23]. Therefore, the controller of each agent must be based only on relative measurements and information acquired through the communication channels from the neighboring agents in the network. In [24], the authors have proposed the neighborhood controller–neighborhood observer protocol for output synchronization of homogeneous networks, while the heterogeneous networks are considered in [23]. The COR problem for linear heterogeneous non-introspective agents is investigated in [25], and for nonlinear agents in [26].

In majority of the existing consensus and COR protocols [12], [13], [14], [16], [17], [18], [20], [21], the internal controller (observer) states are transmitted between agents via the communication network and used as inputs for the distributed observer. Some protocols, such as those in [7], [22], [23], and [25], even require the additional exchange of output or state measurements. Recently, significant research efforts have been devoted to developing consensus and COR protocols that rely solely on the exchange of output measurements. Such protocols can greatly reduce the communication burden since output measurements typically have a lower dimension than controller states [27], [28]. Additionally, when agents can measure their neighbors' relative output information, these protocols can be implemented without establishing the communication network [29], [30].

However, the limited exchange of information between agents gives rise to difficulties in analyzing the stability of the closed-loop system. The dynamics of the distributed observer becomes coupled with that of the heterogeneous multi-agent system, and as a consequence, the controller and observer gains cannot be independently designed. In [31] and [32], the authors developed a low-gain technique based on the small-gain condition to design a distributed observer for networks of introspective agents. Nonetheless, this method cannot guarantee the solvability of the COR problem in the presence of external disturbances, thus limiting its practical applications. More recently, the COR problem without controller state exchange was investigated in [33], and a stability condition based on the dynamics of the overall multi-agent system has been derived. However, the proposed design algorithm does not directly incorporate this condition into the design process. Instead, the controller and observer gains are independently calculated using Riccati equations, and the process is iteratively repeated until the stability condition is fulfilled. Lately, tools from $\mathcal{H}_\infty$ control theory have proven to be of great use for the design of the distributed internal-model-based protocols [34], [35], [36], while their potential in the design of distributed observer-based protocols is not yet fully exploited.

In networks of non-introspective agents, both agent and exosystem states need to be estimated in a distributed manner since local output measurements are not available. Consequently, designing distributed observers without knowledge of neighboring agents' controller states becomes an even more challenging task than it is in networks of introspective agents. In [27] and [28], the authors solve the COR problem for networks of heterogeneous agents with minimum-phase dynamics and identical relative degrees. The tracking problem in homogeneous MASs with general linear dynamics is solved in [29] and [30] by introducing the local observer that estimates synchronization error. Further, the heterogeneous MASs with general linear dynamics have been studied in [37], but no guarantees for the solvability of the COR problem by the proposed design method are established, even for the agents with minimum-phase dynamics.

Motivated by above discussion, we propose a novel distributed observer-based approach that solves the COR problem in heterogeneous linear MASs without requiring agents to exchange the internal controller states. Unlike other papers in the literature which address either the case of non-introspective or introspective agents, here we consider both cases in a unified framework based on tools from $\mathcal{H}_\infty$ control. An extensive solvability analysis is carried out from both the agents' and exosystem's perspective. Under the assumption that the poles of exosystems lie on the imaginary axis, it is proven that the solvability of the COR problem can be guaranteed in advance for: i) introspective agents with arbitrary dynamics, ii) non-introspective agents with stable dynamics.

The main contributions of the paper can be summarized as follows:

1) Contrary to the observer-type protocols [12], [13], [14], [16], [17], [18], [20], [21], [22], [23], [25], the approach proposed in this paper does not require the exchange of the controller states among the agents. Instead, only the output information needs to be shared, which considerably reduces the communication burden. Moreover, if the followers are equipped with sensors that provide them with relative output measurements of the neighboring agents, then the proposed protocols can be implemented without establishing a communication network, making a MAS more secure [30].
2) A novel controller design method that combines the advantages of parametric algebraic Riccati equations (ARE) and tools from $\mathcal{H}_\infty$ theory is devel-

oped. Compared to the low-gain method [31], [32], the proposed approach provides greater flexibility in designing the controller parameters, thus enabling better tuning of the system performance. Moreover, the solvability of the COR problem is guaranteed for introspective linear agents with arbitrary dynamics in the presence of disturbances, which extends results in [31] and [32]. Furthermore, contrary to [33], the stability condition in our paper depends on the individual dynamics of each agent, rather than the dynamics of the overall MAS.

3) In addition to introspective agents, a more challenging case of the COR problem in the networks of non-introspective agents is also considered. Compared to the protocols [29], [30] that are devised for homogeneous agents, in this paper the agents are allowed to be heterogeneous with a general linear dynamics. Furthermore, the existence of a solution to the COR problem is guaranteed for agents with poles in the closed left half-plane, which extends the results in [27] and [28], where heterogeneous agents are assumed to be minimum-phase with an identical relative degree.

The rest of the paper is organized as follows. In Section II, the preliminaries are given and the COR problem is formally stated. In Sections III and IV, COR protocols for non-introspective and introspective agents are proposed, respectively. Section V deals with extensive solvability analysis. Finally, numerical simulations and concluding remarks are given in Sections VI and VII, respectively.

*Notation*: $I$ is an identity matrix with appropriate dimension. For a square matrix $A$, $\lambda(A)$ denotes its spectrum, and $\rho(A)$ its spectral radius. The absolute value of a matrix is defined as $|A| = [|a_{ij}|]$, where $A = [a_{ij}]$. Moreover, $A > 0$ ($\geq 0$) means that $A$ is positive definite (semidefinite). The Kronecker product of two matrices $A$ and $B$ is denoted as $A \otimes B$. For a stable system $G(s)$, $\|G\|_\infty$ denotes its $\mathcal{H}_\infty$ norm, while $\|G(j\omega)\|$ denotes its largest singular value with respect to the frequency. The operator diag$(\cdot)$ builds a (block) diagonal matrix from its arguments.

## II. PRELIMINARIES AND PROBLEM FORMULATION

Consider a multi-agent system consisting of $N$ linear heterogeneous agents described by the following dynamics:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i \omega \\ y_i &= C_i x_i + Q\omega, \ i = 1, \dots, N, \end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the state, $u_i(t) \in \mathbb{R}^{m_i}$ is the control input, $y_i(t) \in \mathbb{R}^p$ is the output of the agent $i$, and $\omega(t) \in \mathbb{R}^{q_\omega}$ is the state of an exosystem $\Sigma_\omega$ that represents disturbance to be rejected. The disturbance is generated as follows

$$\Sigma_\omega : \dot{\omega} = P\omega, \tag{2}$$

where $P$ is a constant matrix. Furthermore, suppose that the reference signal, denoted by $y_0(t)$, is generated by an exosystem

$$\Sigma_\nu : \begin{cases} \dot{\nu} = S\nu \\ y_0 = F\nu \end{cases}, \tag{3}$$

where $\nu(t) \in \mathbb{R}^{q_\nu}$ is the state, and $y_0(t) \in \mathbb{R}^p$ is the output of the exosystem $\Sigma_\nu$. Note that the time index $t$ has been dropped in the equations for sake of clarity.

The $N$ agents with the dynamics (1), called the *followers*, and an agent described by the exosystem (3), termed the *leader*, can be represented by a node set $\mathcal{V} = \{0, 1, 2, \dots, N\}$, with 0 corresponding to the leader. The interactions among the agents are modeled by an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, where the ordered pair $(i, j) \in \mathcal{E}$ indicates the existence of a directed link from node $i$ to node $j$. In such case, we say that the node $i$ is a neighbor of the node $j$. The set of all neighbors of the node $i$ is denoted by $\mathcal{N}_i$. An adjacency matrix associated with a directed graph (digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is denoted as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$, where $a_{ij} > 0$ for $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Without loss of generality, we assume $\sum_{j=0}^{N} a_{ij} = 1, \ i = 0, \dots, N$. Then, Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is defined as $\mathcal{L} = I - \mathcal{A}$. The Laplacian and adjacency matrices can be partitioned in the following way

$$\mathcal{A} = \begin{bmatrix} 1 & 0 \\ a_0 & \bar{\mathcal{A}} \end{bmatrix}, \ \mathcal{L} = \begin{bmatrix} 0 & 0 \\ -a_0 & \bar{\mathcal{L}} \end{bmatrix}, \tag{4}$$

where $a_0 = [a_{10}, a_{20}, \dots, a_{N0}]^T$.

We assume that each follower has access to relative outputs of the neighboring agents, thus obtaining the quantity $\zeta_i = \sum_{j \in \mathcal{N}_i} a_{ij}(y_i - y_j)$ which can be written in terms of the elements of the Laplacian matrix as $\zeta_i = \sum_{j=0}^{N} l_{ij} y_j, \ i = 1, \dots, N$. The regulated output for each agent is defined as follows:

$$\tilde{y}_i = y_i - y_0, \ i = 1, \dots, N. \tag{5}$$

The main objective is to design a distributed dynamic control law that drives the regulated output to zero. Based on the information available to each agent we will distinguish the following two cases.

In the first case, we assume that the agents are non-introspective, i.e. they can only measure relative output of the neighboring agents. The goal is to design a distributed dynamic relative output feedback (ROF) control law, i.e. a control law based on a linear combination of relative output measurements $\zeta_i = \sum_{j=0}^{N} l_{ij} y_j, i = 1, \dots, N$. In the second case, it is assumed that agents are introspective, i.e. each agent can measure its output signal in absolute (global) coordinates $y_i$. In this case, we distinguish the following two scenarios. The former requires exchange of $y_i$ through communication channels among the neighboring agents, while the latter assumes that the introspective agents can also measure the relative outputs, thus eliminating the need for communication. The control law for this case, covering both scenarios, will be called the distributed dynamic output feedback (OF) protocol.

The cooperative output regulation (COR) problem can then be stated as follows.

*Problem (COR):* For the multi-agent system composed of (1), (2), and (3), design ROF (OF) controller such that the closed-loop system satisfies the following the conditions:

1) The origin of the overall closed-loop system is asymptotically stable when $\omega = 0$ and $\nu = 0$.
2) For any initial conditions $\nu(0)$, $\omega(0)$, $x_i(0)$, the regulated output satisfies $\lim_{t\to\infty} \tilde{y}_i(t) = 0$, $i = 1, \ldots, N$.

In order to solve the COR Problem, we will formalize the required assumptions:

*Assumption 1:* The digraph $\mathcal{G}$ contains a directed spanning tree with node 0 as its root.

*Assumption 2:* The matrix $S$ has no strictly stable poles.

*Assumption 3:* The pairs $(A_i, B_i)$, $i = 1, \ldots, N$, are stabilizable.

*Assumption 4.1:* For the ROF protocol, the pairs

$$\left(\begin{bmatrix} -C_i & -Q & F \end{bmatrix}, \begin{bmatrix} A_i & E_i & 0 \\ 0 & P & 0 \\ 0 & 0 & S \end{bmatrix}\right), \; i = 1, \ldots, N$$

are detectable.

*Assumption 4.2:* For the OF protocol, the pairs

$$(F, S), \;\; \left(\begin{bmatrix} C_i & Q \end{bmatrix}, \; \begin{bmatrix} A_i & E_i \\ 0 & P \end{bmatrix}\right), \; i = 1, \ldots, N,$$

are detectable.

*Assumption 5:* The linear matrix equations

$$\begin{cases} \Pi_i^{\omega} P = A_i \Pi_i^{\omega} + B_i \Gamma_i^{\omega} + E_i \\ 0 = C_i \Pi_i^{\omega} + Q \end{cases}, \tag{6a}$$

$$\begin{cases} \Pi_i^{\nu} S = A_i \Pi_i^{\nu} + B_i \Gamma_i^{\nu} \\ 0 = C_i \Pi_i^{\nu} - F \end{cases}, \tag{6b}$$

have solution pairs $(\Pi_i^{\omega}, \Gamma_i^{\omega})$ and $(\Pi_i^{\nu}, \Gamma_i^{\nu})$ for $i = 1, \ldots, N$, respectively.

*Remark 1: Assumption 2 is introduced to avoid the trivial case of strictly stable S, as eigenvalues with negative real parts exponentially decay to zero and do not affect the asymptotic behavior of the closed-loop system. Furthermore, if the COR problem is solved for a linear MAS under Assumption 4.2, then it is also solved when this assumption is violated, as stated in [38]. Assumptions 3-5 are standard in the cooperative output regulation literature [13]. The equations (6) are known as regulator equations, whose solvability is a necessary condition for solving the classical output regulation problem [38]. For the OF protocol, Assumption 4.2 can be ensured under mild conditions if the pairs $(C_i, A_i)$ and $(Q, P)$ are detectable. When there is no disturbance acting on the plant output (i.e., $Q = 0$), Assumption 4.2 reduces to the detectability of the pair $\left(\begin{bmatrix} C_i & 0 \end{bmatrix}, \begin{bmatrix} A_i & E_i \\ 0 & P \end{bmatrix}\right)$, which is always detectable if the pairs $(C_i, A_i)$ and $(E_i, A_i)$ are detectable [39]. For the ROF protocol, Assumption 4.1 is always ensured if the pairs $\left(\begin{bmatrix} C_i & 0 \end{bmatrix}, \begin{bmatrix} A_i & E_i \\ 0 & P \end{bmatrix}\right)$ and $(F, S)$ are detectable, and matrices S and P have no common eigenvalues. If S and P have common eigenvalues, Assumption 4.1 does not hold for single-output agents, while for multi-output agents it can be ensured under mild conditions [40].*

*Remark 2: It is worth noting that, similar to works [17], [22], [33], we consider separate exogenous systems to model the disturbance and reference signal. The disturbance $\omega$ is assumed to be unmeasurable for all agents, while at least one agent has knowledge of the reference signal $y_0$. However, as noted in Remark 1, when exosystems S and P have common eigenvalues and the agents have a single output, Assumption 4.1 will not hold. A possible solution to handle this issue is simply to exclude the common eigenvalues from matrix P, as suggested in [33]. An alternative approach in the literature is to model disturbances and reference signals using a single exosystem such as in [13], [14], [15], and [16]. In this case, at least one agent must have access to the complete exosystem state, including disturbances. It is important to highlight that all the results obtained for the ROF protocol are equally applicable to this scenario. Nevertheless, for the sake of ensuring a straightforward parallel between the ROF and OF protocols, we have adopted the same approach for both introspective and non-introspective agents.*

Prior to presenting the main results, we provide a lemma regarding the spectral radius of the matrix $|\mu I - \bar{\mathcal{L}}|$, where $\mu \in \mathbb{R}$. This matrix plays a crucial role in the stability analysis of MAS.

*Lemma 1: Consider the Laplacian and adjacency matrix partition (4). Then, for any real scalar $\mu$, $\rho\left(|\mu I - \bar{\mathcal{L}}|\right) = |\mu - 1| + \rho\left(\bar{\mathcal{A}}\right)$. Moreover, $\rho\left(\bar{\mathcal{A}}\right) < 1$ if and only if the digraph $\mathcal{G}$ associated with the adjacency matrix $\mathcal{A}$ contains a directed spanning tree with node 0 as a root.*

*Proof:* In order to prove the first statement of the lemma, note that $\bar{\mathcal{L}} = I - \bar{\mathcal{A}}$. Since $\bar{\mathcal{A}}$ is a non-negative matrix with zeros on the main diagonal, it is straightforward to show that $|\mu I - \bar{\mathcal{L}}| = |\mu - 1| I + \bar{\mathcal{A}}$. Therefore, $\lambda_i\left(|\mu I - \bar{\mathcal{L}}|\right) = \lambda_i\left(|\mu - 1| I + \bar{\mathcal{A}}\right) = |\mu - 1| + \lambda_i\left(\bar{\mathcal{A}}\right)$, $i = 1, \ldots, N$. Moreover, according to Perron-Frobenius theorem, the non-negativity of $\bar{\mathcal{A}}$ implies that $\rho(\bar{\mathcal{A}})$ is its eigenvalue, from which follows $\rho(|\mu I - \bar{\mathcal{L}}|) = |\mu - 1| + \rho(\bar{\mathcal{A}})$. This completes the proof of the first statement.

For the second statement, the structure of $\mathcal{A}$ implies that it contains the eigenvalue 1 in addition to the eigenvalues of $\bar{\mathcal{A}}$. The matrix $\mathcal{A}$ is row-stochastic, thus according to (*Lemma* 3.4, [1]), it has a simple eigenvalue $\rho(\mathcal{A}) = 1$ if and only if the digraph $\mathcal{G}$ contains a directed spanning tree with node 0 as the root. Therefore, $\rho(\bar{\mathcal{A}}) < 1$. □

## III. COR IN NETWORKS OF NON-INTROSPECTIVE AGENTS

In this section, the distributed ROF controller is first introduced, which is followed by a discussion on the stability analysis and controller design procedure.

Consider the following ROF controller

$$\begin{bmatrix} \dot{\hat{x}}_i \\ \dot{\hat{\omega}}_i \\ \dot{\hat{\nu}}_i \end{bmatrix} = \begin{bmatrix} A_i & E_i & 0 \\ 0 & P & 0 \\ 0 & 0 & S \end{bmatrix} \begin{bmatrix} \hat{x}_i \\ \hat{\omega}_i \\ \hat{\nu}_i \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix} u_i + \begin{bmatrix} L_i^x \\ L_i^{\omega} \\ L_i^{\nu} \end{bmatrix} \varepsilon_i,$$

$$u_i = K_i^x \hat{x}_i + K_i^{\omega} \hat{\omega}_i + K_i^{\nu} \hat{\nu}_i, \; i = 1, \ldots, N, \tag{7}$$

where $\hat{x}_i \in \mathbb{R}^{n_i}$, $\hat{\omega}_i \in \mathbb{R}^{q_\omega}$ and $\hat{v}_i \in \mathbb{R}^{q_v}$ are local estimates of $x_i$, $\omega$ and $v$, respectively. The term $\varepsilon_i \in \mathbb{R}^p$ represents the virtual error signal defined as

$$\varepsilon_i \triangleq \sum_{j=0}^{N} l_{ij} y_j - \mu(C_i\hat{x}_i + Q\hat{\omega}_i - F\hat{v}_i),\ i = 1, \ldots, N, \quad (8)$$

where $\mu$ is a real scalar. The first equation in (7) can be viewed as a distributed observer of the system and exosystems states. The observer gains $L_i^x$, $L_i^\omega$, $L_i^v$ and the control law gains $K_i^x$, $K_i^\omega$, $K_i^v$ are the parameters to be designed.

*Remark 3: In addition to the relative measurements, the virtual error signal contains a term $\mu(C_i\hat{x}_i + Q\hat{\omega}_i - F\hat{v}_i)$ that depends on the local estimates of the system and exosystems states. This implies that the implementation of controller (7) does not require communication among agents. It should be noted that the added term is crucial for the stabilization of MAS, which will be shown later.*

Let $K_i^\omega$ and $K_i^v$ be designed as follows:

$$K_i^\omega = \Gamma_i^\omega - K_i^x \Pi_i^\omega,\ K_i^v = \Gamma_i^v - K_i^x \Pi_i^v. \quad (9)$$

Define the tracking and estimation errors as $e_i = x_i - \Pi_i^\omega \omega - \Pi_i^v v$, $\tilde{x}_i = \hat{x}_i - x_i$, $\tilde{\omega}_i = \hat{\omega}_i - \omega$, $\tilde{v}_i = \hat{v}_i - v$. By taking into account the regulator equations (6), the error dynamics of each subsystem can be written as

$$\begin{bmatrix} \dot{e}_i \\ \dot{\tilde{x}}_i \\ \dot{\tilde{\omega}}_i \\ \dot{\tilde{v}}_i \end{bmatrix} = \begin{bmatrix} A_i + B_iK_i^x & B_iK_i^x & B_iK_i^\omega & B_iK_i^v \\ 0 & A_i & E_i & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & S \end{bmatrix} \begin{bmatrix} e_i \\ \tilde{x}_i \\ \tilde{\omega}_i \\ \tilde{v}_i \end{bmatrix} + \begin{bmatrix} 0 \\ L_i^x \\ L_i^\omega \\ L_i^v \end{bmatrix} \varepsilon_i.$$

Furthermore, let $\tilde{e}_i = \begin{bmatrix} \tilde{x}_i^T & \tilde{\omega}_i^T & \tilde{v}_i^T \end{bmatrix}^T$ and define variable $\xi_i = \sum_{j=1}^{N} l_{ij} C_j e_j - \mu C_i e_i$. Then, $\varepsilon_i$ can be expressed as $\varepsilon_i = \xi_i + \mu \begin{bmatrix} -C_i & -Q & F \end{bmatrix} \tilde{e}_i$. Introduce the following matrices

$$H_i = \begin{bmatrix} A_i & E_i & 0 \\ 0 & P & 0 \\ 0 & 0 & S \end{bmatrix},\ G_i = \begin{bmatrix} -C_i & -Q & F \end{bmatrix},$$

$$K_i = \begin{bmatrix} K_i^x & K_i^\omega & K_i^v \end{bmatrix},\ L_i = \begin{bmatrix} L_i^x \\ L_i^\omega \\ L_i^v \end{bmatrix},$$

which in turn gives the closed-loop dynamics

$$\begin{bmatrix} \dot{e}_i \\ \dot{\tilde{e}}_i \end{bmatrix} = \begin{bmatrix} A_i + B_iK_i^x & B_iK_i \\ 0 & H_i + \mu L_iG_i \end{bmatrix} \begin{bmatrix} e_i \\ \tilde{e}_i \end{bmatrix} + \begin{bmatrix} 0 \\ L_i \end{bmatrix} \xi_i. \quad (10)$$

Denote

$$\begin{aligned} &\phi = col\{\phi_i\},\ (\phi_i = e_i, \tilde{e}_i, \xi_i) \\ &\Phi = diag\{\Phi_i\},\ (\Phi_i = A_i, B_i, C_i, H_i, G_i, K_i^x, K_i, L_i) \\ &\bar{\mathcal{L}} = \tilde{\mathcal{L}} \otimes I_p,, \end{aligned} \quad (11)$$

then the overall system dynamics becomes $\begin{bmatrix} \dot{e} \\ \dot{\tilde{e}} \end{bmatrix} = A_{CL} \begin{bmatrix} e \\ \tilde{e} \end{bmatrix}$, where the closed-loop state matrix is

$$A_{CL} = \begin{bmatrix} A + BK^x & BK \\ L(\bar{\mathcal{L}} - \mu I)C & H + \mu LG \end{bmatrix}. \quad (12)$$

Note that due to (6b), $\tilde{y}_i$ can be written as $\tilde{y}_i = C_ix_i + Q\omega - Fv = C_ie_i$. It can be concluded that $\lim_{t\to\infty} e_i(t) = 0$ implies $\lim_{t\to\infty} \tilde{y}_i(t) = 0$. Therefore, ensuring that $A_{CL}$ is Hurwitz stable is equivalent to solving the COR problem.

### A. STABILITY ANALYSIS

In this subsection, we derive local stability conditions that, if satisfied, ensure the stability of the closed-loop matrix $A_{CL}$.

Define the following matrices

$$\hat{A}_i = \begin{bmatrix} A_i + B_iK_i^x & B_iK_i \\ 0 & H_i + \mu L_iG_i \end{bmatrix},\ \hat{B}_i = \begin{bmatrix} 0 \\ L_i \end{bmatrix},\ \hat{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}, \quad (13)$$

with the corresponding transfer function

$$T_i(s) = \hat{C}_i(sI - \hat{A}_i)^{-1}\hat{B}_i,\ i = 1, \ldots, N. \quad (14)$$

*Theorem 1: Consider a multi-agent system composed of (1), (2) and (3). Then, under the Assumptions 1-5, the ROF protocol (7) solves the COR problem if the following condition holds*

$$\|T_i\|_\infty < \gamma^*,\ i = 1, \ldots, N, \quad (15)$$

*where* $\gamma^* = \frac{1}{\rho(|\mu I - \bar{\mathcal{L}}|)}$.

*Proof: The closed-loop system matrix $A_{CL}$ in (12) can be rewritten as $A_{CL} = \hat{A} + \hat{B}(\bar{\mathcal{L}} - \mu I)\hat{C}$, where*

$$\hat{A} = \begin{bmatrix} A + BK^x & BK \\ 0 & H + \mu LG \end{bmatrix},\ \hat{B} = \begin{bmatrix} 0 \\ L \end{bmatrix},\ \hat{C} = \begin{bmatrix} C & 0 \end{bmatrix}. \quad (16)$$

*Assume that $\hat{A}$ is Hurwitz, which can always be achieved under the Assumptions 3 and 4.1. Then, the matrix determinant lemma gives*

$$\begin{aligned} &\det(sI - A_{CL}) \\ &= \det(sI - \hat{A})\det(I - (sI - \hat{A})^{-1}\hat{B}(\bar{\mathcal{L}} - \mu I)\hat{C}) \\ &= \det(sI - \hat{A})\det(I + (\mu I - \bar{\mathcal{L}})\hat{C}(sI - \hat{A})^{-1}\hat{B}). \end{aligned}$$

*Note that $A_{CL}$ is Hurwitz stable if $\det(sI - A_{CL}) \neq 0$, $\forall s \in \bar{\mathbb{C}}^+$. This means that the matrix $I + (\mu I - \bar{\mathcal{L}})\hat{C}(sI - \hat{A})^{-1}\hat{B}$ must not have zero eigenvalues for any $s \in \bar{\mathbb{C}}^+$, otherwise its determinant will be equal to zero. Therefore, $A_{CL}$ is stable if*

$$\rho((\mu I - \bar{\mathcal{L}})\hat{C}(sI - \hat{A})^{-1}\hat{B}) < 1,\ \forall s \in \bar{\mathbb{C}}^+. \quad (17)$$

*Taking (13) and (16) into account, the transfer function $\hat{C}(sI - \hat{A})^{-1}\hat{B}$ can be expressed as*

$$\begin{aligned} &\hat{C}(sI - \hat{A})^{-1}\hat{B} \\ &= C(sI - (A + BK^x))^{-1}BK(sI - (H + \mu LG))^{-1}L \\ &= diag(C_i(sI - (A_i + B_iK_i^x))^{-1}B_iK_i(sI - (H_i + \mu L_iG_i))^{-1}L_i) \\ &= \mathrm{diag}(T_i(s)). \end{aligned}$$

*Furthermore, by using the block-norm matrix inequality [34] we can write*

$$\rho((\mu I - \bar{\mathcal{L}})\mathrm{diag}(T_i(s))) \leq \rho(|\mu I - \bar{\mathcal{L}}|\mathrm{diag}(\|T_i\|_\infty)).$$

*According to Lemma 8 in [34], the inequality* $\rho((\mu I - \bar{\mathcal{L}})\,\mathrm{diag}(\|T_i\|_\infty)) \leq \rho(|\mu I - \bar{\mathcal{L}}|)\max_i \|T_i\|_\infty$ *holds, which allows the stability condition to be written as*

$$\rho(|\mu I - \bar{\mathcal{L}}|)\max_i \|T_i\|_\infty < 1.$$

*The above condition is equivalent to the condition (15), which completes the proof.* □

*Remark 4: It should be noted that the proposed controller relies on knowledge of a global information and is not fully distributed. Actually, most of the existing protocols, for instance [28], [31], [32], [34], [35], [36], also rely on knowledge of global information. It is worth noting that in many practical situations, it is possible to know or predict the lower bound of a spectral radius of* $|\mu I - \bar{\mathcal{L}}|$. *One possible approach to develop a fully distributed protocol is by employing adaptive gain methods, as demonstrated in [2] and [29].*

In the following lemma, we establish the lower bound for $\|T_i\|_\infty$.

*Lemma 2: Suppose that the transfer function* $T_i(s)$ *is stable. Then,* $\mu^{-1}$ *is a lower bound of* $\|T_i\|_\infty$, *i.e.*

$$\|T_i\|_\infty \geq \frac{1}{\mu},\ i = 1, \ldots, N. \tag{18}$$

*Proof:* Introduce the following coordinate transformation matrix

$$M_i = \begin{bmatrix} I & I & -\Pi_i^\omega & -\Pi_i^\nu \\ 0 & I & -\Pi_i^\omega & -\Pi_i^\nu \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}.$$

In the new state coordinates, the systems matrices become $\check{A}_i = M_i \hat{A}_i M_i^{-1}$, $\check{B}_i = M_i \hat{B}_i$ and $\check{C}_i = \hat{C}_i M_i^{-1}$. Taking into account (9), it can be shown that these matrices are

$$\check{A}_i = \begin{bmatrix} A_i + B_i K_i^x & -\mu\,[\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]\,L_i C_i & 0 & 0 \\ 0 & A_i - \mu\,[\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]\,L_i C_i & -B_i\Gamma_i^\omega & -B_i\Gamma_i^\nu \\ 0 & -\mu L_i^\omega C_i & P & 0 \\ 0 & -\mu L_i^\nu C_i & 0 & S \end{bmatrix},$$

$$\check{B}_i = \begin{bmatrix} [\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]L_i \\ [\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]L_i \\ L_i^\omega \\ L_i^\nu \end{bmatrix},\ \check{C}_i = \begin{bmatrix} C_i & -C_i & 0 & 0 \end{bmatrix}. \tag{19}$$

Suppose that $\|T_i\|_\infty < \mu^{-1}$ holds. Then, by the means of the small-gain theorem, the feedback controller $\check{u}_i = -\mu \check{y}_i$ stabilizes the system (19). The resulting state matrix is then equal to

$$\check{A}_i - \mu \check{B}_i \check{C}_i = \begin{bmatrix} A_i + B_i K_i^x - \mu[\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]L_i C_i & 0 & 0 & 0 \\ -\mu[\,I\ -\Pi_i^\omega\ -\Pi_i^\nu\,]L_i C_i & A_i & -B_i\Gamma_i^\omega & -B_i\Gamma_i^\nu \\ -\mu L_i^\omega C_i & 0 & P & 0 \\ -\mu L_i^\nu C_i & 0 & 0 & S \end{bmatrix}.$$

However, since $\lambda(\check{A}_i - \mu\check{B}_i\check{C}_i) = \lambda(A_i) \cup \lambda(P) \cup \lambda(S) \cup \lambda(A_i + B_i K_i^x - \mu(L_i^x - \Pi_i^\omega L_i^\omega - \Pi_i^\nu L_i^\nu)C_i)$, it can be concluded that it is impossible to stabilize the system with the controller $\check{u}_i = -\mu\check{y}_i$. Thus, (18) must hold. □

So far we have not justified the necessity of the Assumption 1. This will be done in the following corollary.

*Corollary 1: The stability condition (15) can be satisfied if and only if Assumption 4.1 holds and*

$$\mu > \frac{1 + \rho(\bar{A})}{2}. \tag{20}$$

*Proof:* In order for the condition (15) to be satisfiable, it is clear that $\gamma^*$ must be greater than the lower bound of $\|T_i\|_\infty$, i.e. the following must hold

$$\rho(|\mu I - \bar{\mathcal{L}}|) < \mu. \tag{21}$$

*If part*: According to *Lemma 1*, if Assumption 4.1 holds, then $\rho\left(\left|\mu I - \bar{\mathcal{L}}\right|\right) = |\mu - 1| + \rho\left(\bar{A}\right)$ and $\rho(\bar{A}) < 1$. Therefore, (21) can be rewritten as $|\mu - 1| + \rho(\bar{A}) < \mu$. It can be easily checked that this inequality holds for any $\mu$ that satisfies (20).

*Only if part*: Suppose that Assumption 4.1 does not hold. Then, from *Lemma 1* it follows that $\rho(\bar{A}) = 1$. Therefore, $\rho\left(\left|\mu I - \bar{\mathcal{L}}\right|\right) = |\mu - 1| + 1$, which is always greater or equal to $\mu$. □

### B. CONTROL LAW SYNTHESIS

Generally, it is difficult to find $K_i$ and $L_i$ such that $\|T_i\|_\infty < \gamma^*$, since the gain matrix $L_i$ is embedded in both the state and input matrix, thus a general $\mathcal{H}_\infty$ design method cannot be applied. Instead, we will first determine the gain $L_i$ such that $H_i + \mu L_i G_i$ is stable. Note that it is always possible to find such $L_i$ since the pair $(H_i, G_i)$ is detectable by Assumption 4.1. After $L_i$ is determined, the problem reduces to finding the gain $K_i^x$ such that (15) holds, since $K_i^\omega = \Gamma_i^\omega - K_i^x \Pi_i^\omega$, $K_i^\nu = \Gamma_i^\nu - K_i^x \Pi_i^\nu$.

The problem of determining the gain matrix $L_i$ such that $H_i + \mu L_i G_i$ is Hurwitz stable is well-studied in the traditional control literature. In this paper, we introduce the following algebraic parametric Ricatti equation (ARE)

$$X_i(\epsilon_i)H_i^T + H_i X_i(\epsilon_i) - \delta_i X_i(\epsilon_i) G_i^T G_i X_i(\epsilon_i) + \epsilon_i I = 0, \tag{22}$$

where $\delta_i$ is a small positive constant and $\epsilon_i > 0$ is an adjustable parameter. After solving (22) for $X_i(\epsilon_i)$, the gain $L_i$ is calculated as $L_i = -\mu^{-1} X_i(\epsilon_i) G_i^T$.

For the fixed $L_i$, the problem can be converted to the standard $\mathcal{H}_\infty$ static output feedback (SOF) problem. Namely, the state matrix $\hat{A}_i$ can be rewritten as $\hat{A}_i = \bar{A}_i + \bar{B}_i K_i^x \bar{C}_i$, where

$$\bar{A}_i = \begin{bmatrix} A_i & \begin{bmatrix} 0 & B_i\Gamma_i^\omega & B_i\Gamma_i^\nu \end{bmatrix} \\ 0 & H_i + \mu L_i G_i \end{bmatrix}, \tag{23}$$

$$\bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix},\ \bar{C}_i = \begin{bmatrix} I & \begin{bmatrix} I & -\Pi_i^\omega & -\Pi_i^\nu \end{bmatrix} \end{bmatrix}. \tag{24}$$

The task is to find the output feedback gain $K_i^x$ such that the transfer function $T_i(s) = \hat{C}_i(sI - \bar{A}_i - \bar{B}_i K_i^x \bar{C}_i)^{-1}\hat{B}_i$ is stable and $\|T_i\|_\infty < \gamma_i$, for $i = 1, \ldots, N$. An additional variable $\gamma_i \leq \gamma^*$ has been introduced, which gives more freedom for setting the upper bound of $\|T_i\|_\infty$. Necessary condition

**Algorithm 1** $\mathcal{H}_\infty$ Static Output Feedback (SOF) Algorithm

1: Set $\mathcal{P}_0 = I$ and $\mathcal{Q}_0 = I$

2: Solve the following optimization problem for $\mathcal{P}, \mathcal{Q}, \mathcal{V}_1, \mathcal{V}_2$

$$\min \quad \text{trace}(\mathcal{P}\mathcal{Q}_0 - \mathcal{Q}\mathcal{P}_0), \text{ s.t. constraints}$$
$$\begin{pmatrix} \mathcal{P}\bar{A}_i + \bar{A}_i^T\mathcal{P} + \mathcal{V}_1\bar{C}_i + \bar{C}_i^T\mathcal{V}_1^T & \mathcal{P}\hat{B}_i & \hat{C}_i^T \\ \hat{B}_i^T\mathcal{P} & -\gamma_i I & 0 \\ \hat{C}_i & 0 & -\gamma_i I \end{pmatrix} < 0$$
$$\begin{pmatrix} \bar{A}_i\mathcal{Q} + \mathcal{Q}\bar{A}_i^T + \bar{B}_i\mathcal{V}_2 + \mathcal{V}_2^T\bar{B}_i^T & \hat{B}_i & \hat{C}_i^T\mathcal{Q} \\ \hat{B}_i^T & -\gamma_i I & 0 \\ \mathcal{Q}\hat{C}_i & 0 & -\gamma_i I \end{pmatrix} < 0$$
$$\begin{pmatrix} \mathcal{P} & I \\ I & \mathcal{Q} \end{pmatrix} \geq 0, \mathcal{P} > 0, \mathcal{Q} > 0$$

3: Check the following conditions:
   1) if $\text{trace}(\mathcal{P}\mathcal{Q}) - n < \varepsilon_1$, a prescribed tolerance, go to Step 4
   2) if $\text{trace}(\mathcal{P}\mathcal{Q}) - \text{trace}(\mathcal{P}_0\mathcal{Q}_0) < \varepsilon_2$, a prescribed tolerance, initial $\mathcal{P}$ may not be found, EXIT
   3) otherwise set $\mathcal{P}_0 = \mathcal{P}$, $\mathcal{Q}_0 = \mathcal{Q}$ and go to Step 2.

4: Set $\mathcal{P}_0 = \mathcal{P}$. Solve the following optimization problem for $K_i^x$ with given $\mathcal{P}$

$$\min \quad \alpha, \text{ s.t. constraint}$$
$$\Phi = \mathcal{P}\bar{A}_i + \bar{A}_i^T\mathcal{P} + \mathcal{P}\bar{B}_iK_i^x\bar{C}_i + \bar{C}_i^TK_i^{xT}\bar{B}_i^T\mathcal{P} - \alpha\mathcal{P}$$
$$\begin{pmatrix} \Phi & \mathcal{P}\hat{B}_i & \hat{C}_i^T \\ \hat{B}_i^T\mathcal{P} & -\gamma_i I & 0 \\ \hat{C}_i & 0 & -\gamma_i I \end{pmatrix} < 0$$

5: if $\alpha \leq 0$, the stabilizing $K_i^x$ is found, EXIT

6: Solve the following optimization for $\mathcal{P}$ problem with given $K_i^x$:

$$\min \quad \alpha, \text{ s.t constraint from the Step 4.}$$

7: if $\alpha \leq 0$, the stabilizing $K_i^x$ is found, EXIT

8: Solve the following optimization problem for $\mathcal{P}$ with given $K_i^x$ and $\alpha$:

$$\min \text{trace}(\mathcal{P}) \quad \text{s.t constraint from the Step 4.}$$

9: Check the following conditions:
   1) if $\|\mathcal{P} - \mathcal{P}_0\| / \|\mathcal{P}\| < \delta$, the solution may not exists, EXIT
   2) otherwise go to Step 4

for solving the $\mathcal{H}_\infty$ SOF problem is that Assumption 3 holds.

There are many available algorithms in the literature for solving the $\mathcal{H}_\infty$ SOF problem [41], [42], [43]. In this paper, we have adopted the iterative LMI (ILMI) approach developed in [43], which utilizes a separate algorithm to optimize initial values of the variables and thus is very effective in finding the solution. The ILMI method is presented in *Algorithm* 1, where the notation is adapted to one used in this paper.

**Algorithm 2** Design of ROF Controllers

1: Set $\mu$ according to Corollary 1. Initialize $\epsilon_i$ and $\delta_i$.

2: Compute $L_i = -\mu^{-1}X_i(\epsilon_i)G_i^T$, where $X_i(\epsilon_i)$ is the solution of the ARE:

$$X_i(\epsilon_i)H_i^T + H_iX_i(\epsilon_i) - \delta_iX_i(\epsilon_i)G_i^TG_iX_i(\epsilon_i) + \epsilon_iI = 0 \quad (25)$$

3: Compute the gain $K_i^x$ by Algorithm 1.

4: If the Algorithm 1 does not return the stabilizing solution, return to Step 1 and decrease $\epsilon_i$. If the Algorithm 1 returns the stabilizing solution, but performance is not satisfactory, return to Step 1 and increase $\epsilon_i$.

Further discussion regarding the solvability and important properties of ARE (22) will be provided in Section V. It should be noted that although fixing the gain $L_i$ generally reduces the freedom for determining $K_i^x$, it will be shown that the proposed approach always ensures the solvability of the COR problem for certain types of plants and exosystems.

The complete procedure for determining the gains $L_i$ and $K_i^x$ is summarized in *Algorithm 2*.

## IV. COR IN NETWORKS OF INTROSPECTIVE AGENTS

This section presents a distributed OF controller and discusses the MAS stability, as well as the controller design procedure.

Consider the following OF controller

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}}_i \\ \dot{\hat{\omega}}_i \end{bmatrix} &= \begin{bmatrix} A_i & E_i \\ 0 & P \end{bmatrix} \begin{bmatrix} \hat{x}_i \\ \hat{\omega}_i \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i \\ &\quad + \begin{bmatrix} L_i^x \\ L_i^\omega \end{bmatrix} \left( y_i - \begin{bmatrix} C_i & Q \end{bmatrix} \begin{bmatrix} \hat{x}_i \\ \hat{\omega}_i \end{bmatrix} \right), \\ \dot{\hat{v}}_i &= S\hat{v}_i + L_i^v\varepsilon_i, \\ u_i &= K_i^x\hat{x}_i + K_i^\omega\hat{\omega}_i + K_i^v\hat{v}_i, \ i = 1, \ldots, N, \end{aligned} \quad (26)$$

where $\hat{x}_i \in \mathbb{R}^{n_i}$, $\hat{\omega}_i \in \mathbb{R}^{q_\omega}$ and $\hat{v}_i \in \mathbb{R}^{q_v}$ are the local estimates of $x_i$, $\omega$ and $v$, respectively. The observer gains $L_i^x$, $L_i^\omega$, $L_i^v$ and the control law gains $K_i^x$, $K_i^\omega$, $K_i^v$ are the parameters to be designed.

In the case of the introspective agents, the virtual error signal $\varepsilon_i \in \mathbb{R}^p$ is defined as

$$\varepsilon_i \triangleq \sum_{j=0}^{N} l_{ij}y_j - \mu(y_i - F\hat{v}_i), \ i = 1, \ldots, N, \quad (27)$$

where $\mu$ is a real scalar. Note that the additional term in equation (27) is different from that in equation (8) since the $i$th agent has access to its own output $y_i$.

*Remark 5: The availability of the local output $y_i$ allows for the design of more efficient control strategies for the introspective agents. In (26), the reference signal is estimated by a distributed observer using a virtual error signal, while the disturbance and system states are estimated based on locally available information. This approach substantially differs from the approaches [31], [32], where both the reference and disturbance signals are estimated by a distributed observer based on information diffused through the network. The benefit of constructing a local observer is that it facilitates the design of the observer gains.*

Let $K_i^{\omega}$ and $K_i^{\nu}$ be designed in the same way as in (9). Define the tracking error, local observer error and distributed observer error as

$$e_i = x_i - \Pi_i^{\omega}\omega - \Pi_i^{\nu}\nu,\ \tilde{e}_i = \begin{bmatrix} \tilde{x}_i \\ \tilde{\omega}_i \end{bmatrix} = \begin{bmatrix} \hat{x}_i - x_i \\ \hat{\omega}_i - \omega \end{bmatrix},$$
$$\tilde{\nu}_i = \hat{\nu}_i - \nu.$$

Substituting $e_i$, $\tilde{e}_i$, $\tilde{\nu}_i$ into (26) yields the following closed-loop dynamics

$$\begin{bmatrix} \dot{e}_i \\ \dot{\tilde{e}}_i \\ \dot{\tilde{\nu}}_i \end{bmatrix} = \begin{bmatrix} A_i + B_iK_i^x & B_iK_i^{x\omega} & B_iK_i^{\nu} \\ 0 & H_i + L_i^{x\omega}G_i & 0 \\ 0 & 0 & S + \mu L_i^{\nu}F \end{bmatrix} \begin{bmatrix} e_i \\ \tilde{e}_i \\ \tilde{\nu}_i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ L_i^{\nu} \end{bmatrix} \xi_i.$$

Similarly to the procedure in the previous section, define $\xi_i = \sum_{j=1}^{N} l_{ij}C_je_j - \mu C_ie_i$ and

$$H_i = \begin{bmatrix} A_i & E_i \\ 0 & P \end{bmatrix},\ G_i = \begin{bmatrix} C_i & Q \end{bmatrix},$$
$$K_i^{x\omega} = \begin{bmatrix} K_i^x & K_i^{\omega} \end{bmatrix},\ L_i^{x\omega} = \begin{bmatrix} L_i^x \\ L_i^{\omega} \end{bmatrix}.$$

Following the rule (11) for notations, the closed-loop state matrix can be written as

$$A_{CL} = \begin{bmatrix} A + BK^x & BK^{x\omega} & BK^{\nu} \\ 0 & H + L^{x\omega}G & 0 \\ L^{\nu}(\tilde{\mathcal{L}} - \mu I)C & 0 & \tilde{S} + \mu L^{\nu}\tilde{F} \end{bmatrix}, \quad (28)$$

where $\tilde{S} = I_N \otimes S$, $\tilde{F} = I_N \otimes F$. Under the same arguments as in the previous section, it can be concluded that $A_{CL}$ being Hurwitz is equivalent to solving the COR problem.

### A. STABILITY ANALYSIS

Let us first define $\bar{A}_{CL} = \begin{bmatrix} A+BK^x & BK^{\nu} \\ L^{\nu}(\tilde{\mathcal{L}}-\mu I)C & \tilde{S}+\mu L^{\nu}\tilde{F} \end{bmatrix}$. Then, the following lemma is of particular importance for the stability analysis.

*Lemma 3: The closed-loop state matrix $A_{CL}$ in (28) is stable if and only if the matrices $H+L^{x\omega}G$ and $\bar{A}_{CL}$ are stable.*

*Proof:* The proof follows from the similarity relation

$$A_{CL} \sim \begin{bmatrix} H + L^{x\omega}G & 0 \\ \begin{matrix} BK^{x\omega} \\ 0 \end{matrix} & \bar{A}_{CL} \end{bmatrix},$$

where the structure of the matrix allows the separation principle to be applied. □

Under the Assumption 4.2, it is always possible to find the gain matrix $L_i^{x\omega}$ such that $H_i + L_i^{x\omega}G_i$ is Hurwitz stable for $i = 1, \ldots, N$, which further implies stability of the matrix $H + L^{x\omega}G$.

In order to analyze stability properties of $\bar{A}_{CL}$, let us first introduce the matrices

$$\hat{A}_i = \begin{bmatrix} A_i + B_iK_i^x & B_iK_i^{\nu} \\ 0 & S + \mu L_i^{\nu}F \end{bmatrix},\ \hat{B}_i = \begin{bmatrix} 0 \\ L_i^{\nu} \end{bmatrix},\ \hat{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}, \quad (29)$$

with the corresponding transfer function

$$T_i(s) = \hat{C}_i(sI - \hat{A}_i)^{-1}\hat{B}_i. \quad (30)$$

Then, we present the stability condition for the multi-agent system under the OF protocol.

*Theorem 2: Consider a multi-agent composed of (1), (2) and (3). Then, if $H_i + L_i^{x\omega}G_i$ is Hurwitz stable and Assumptions 1-5 hold, the OF protocol (26) solves the COR problem provided that the following condition holds*

$$\|T_i\|_{\infty} < \gamma^*,\ i = 1, \ldots, N, \quad (31)$$

where $\gamma^* = \frac{1}{\rho(|\mu I - \hat{\mathcal{L}}|)}$.

*Proof:* Given the Hurwitzness of $H_i + L_i^{x\omega}G_i$, it remains to ensure the stability of $\bar{A}_{CL}$ according to *Lemma 3*. The stability analysis of $\bar{A}_{CL}$ is analogous to the proof of *Theorem 1*, with respect to the newly defined matrices $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$ and the corresponding transfer function $T_i(s) = \hat{C}_i(sI - \hat{A}_i)^{-1}\hat{B}_i$. □

Similarly to the previous section, we establish the lower bound for $||T_i||_{\infty}$.

*Lemma 4: Suppose that the transfer function $T_i(s)$ is stable. Then, $\mu^{-1}$ is a lower bound of $\|T_i\|_{\infty}$, i.e.*

$$\|T_i\|_{\infty} \geq \frac{1}{\mu},\ i = 1, \ldots, N, \quad (32)$$

*Proof:* Introduce the following coordinate transformation matrix

$$M_i = \begin{bmatrix} I & -\Pi_i^{\nu} \\ 0 & I \end{bmatrix}.$$

Then, the new system matrices can be expressed as $\check{A}_i = M_i\hat{A}_iM_i^{-1}$, $\check{B}_i = M_i\hat{B}_i$ and $\check{C}_i = \hat{C}_iM_i^{-1}$, that is

$$\check{A}_i = \begin{bmatrix} A_i + B_iK_i^x & -\mu\Pi_i^{\nu}L_i^{\nu}F \\ 0 & S + \mu L_i^{\nu}F \end{bmatrix},$$
$$\check{B}_i = \begin{bmatrix} -\Pi_i^{\nu}L_i^{\nu} \\ L_i^{\nu} \end{bmatrix},\ \check{C}_i = \begin{bmatrix} C_i & F \end{bmatrix}. \quad (33)$$

The rest of the proof is analogous to the proof of *Lemma 2*. □

*Corollary 2: The stability condition (31) can be satisfied if and only if Assumption 4.1 holds and $\mu > \frac{1+\rho(\bar{\mathcal{A}})}{2}$.*

*Proof:* The proof is analogous to the proof of *Corollary 1*. □

*Remark 6: A special type of graphs that are extensively investigated in the literature of cooperative control are* *acyclic graphs [44]. Acyclic graphs are characterized by a lower triangular matrix $\bar{\mathcal{L}}$ with ones on the main diagonal. The consequence of this structure is that for $\mu = 1$ one gets $\gamma^* \to \infty$, which means that for acyclic graphs both the ROF and OF protocols solve the COR problem with a sufficient condition being that local transfer functions are stable.*

### B. CONTROLLER SYNTHESIS

In this subsection, an algorithm for synthesis of the OF controller is provided. Here we encounter the same difficulty as in the case of the ROF protocol, which is the simultaneous calculation of $K_i$ and $L_i^v$. Therefore, we first find the gains $L_i^{x\omega}$ and $L_i^v$ such that $H_i + L_i^{x\omega}G_i$ and $S + \mu L_i^v F$ are Hurwitz stable, which is always possible under Assumption 4.2. Then, under Assumption 3, we will determine $K_i^x$ such that $A_i + B_i K_i^x$ is stable and $\|T_i\|_\infty < \gamma_i$, where $\gamma_i \leq \gamma^*$.

Observe that, according to *Lemma 3*, the eigenvalues of the matrix $H_i + L_i^{x\omega}G_i$ can be assigned independently of the remaining eigenvalues of the matrix $A_{CL}$. Therefore, $L_i^{x\omega}$ can be obtained using standard ARE-based methods. In this paper, we use the following ARE

$$Y_i(\kappa_i)H_i^T + H_i Y_i(\kappa_i) - Y_i(\kappa_i)G_i^T G_i Y_i(\kappa_i) + \kappa_i I = 0, \quad (34)$$

which always has a solution for $\kappa_i > 0$ under Assumption 4.2 [45]. The stabilizing gain is then computed as $L_i^{x\omega} = -Y_i(\kappa_i)G_i^T$.

On the other hand, the gain $L_i^v$ is computed as $L_i^v = -\mu^{-1}X_i(\epsilon_i)F^T$, where $X_i(\epsilon_i)$ is the solution of the ARE introduced in the Section III-B:

$$X_i(\epsilon_i)S^T + SX_i(\epsilon_i) - \delta_i X_i(\epsilon_i)F^T F X_i(\epsilon_i) + \epsilon_i I = 0. \quad (35)$$

In the previous equation, $\delta_i$ represents a small positive constant, and $\epsilon_i > 0$ is an adjustable parameter.

Under the OF protocol, the state matrix $\hat{A}_i$ can be rewritten as $\hat{A}_i = \bar{A}_i + \bar{B}_i K_i^x \bar{C}_i$, where

$$\bar{A}_i = \begin{bmatrix} A_i & B_i\Gamma_i^v \\ 0 & S + \mu L_i^v F \end{bmatrix}, \quad (36)$$

$$\bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \ \bar{C}_i = \begin{bmatrix} I & -\Pi_i^v \end{bmatrix}. \quad (37)$$

Therefore, $K_i^x$ can be determined by using the *Algorithm* 1. The other feedforward gains are computed as $K_i^\omega = \Gamma_i^\omega - K_i^x\Pi_i^\omega$ and $K_i^v = \Gamma_i^v - K_i^x\Pi_i^v$.

The complete procedure for control synthesis is summarized in *Algorithm* 3. Further discussion regarding the solvability of the COR problem by the proposed approach is provided in Section V.

## V. SOLVABILITY ANALYSIS

In this section, under some additional assumptions for leader's dynamics, we provide further analysis regarding the solvability of the COR problem. It is shown that under the ROF protocol, the existence of a solution to the COR problem can be guaranteed whenever $\lambda(A_i) \in \mathbb{C}^-$, $i = 1, \ldots, N$. Furthermore, under the OF protocol, a solution to the COR problem always exists.

**Algorithm 3** Design of OF Controllers

1: Set $\mu$ according to Corollary 2. Initialize $\kappa_i$,$\epsilon_i$ and $\delta_i$.

2: Compute $L_i^{x\omega} = -Y_i(\kappa_i)G_i^T$, where $Y_i(\kappa_i)$ is the solution of the ARE:

$$Y_i(\kappa_i)H_i^T + H_i Y_i(\kappa_i) - Y_i(\kappa_i)G_i^T G_i Y_i(\kappa_i) + \kappa_i I = 0 \quad (38)$$

3: Compute $L_i^v = -\mu^{-1}X_i(\epsilon_i)F^T$, where $X_i(\epsilon_i)$ is the solution of the ARE:

$$X_i(\epsilon_i)S^T + SX_i(\epsilon_i) - \delta_i X_i(\epsilon_i)F^T F X_i(\epsilon_i) + \epsilon_i I = 0, \quad (39)$$

4: Compute the gain $K_i^x$ by Algorithm 1.

5: If the Algorithm 1 does not return the stabilizing solution, return to Step 1 and decrease $\epsilon_i$. If the Algorithm 1 returns stabilizing solution, but performance is not satisfactory, return to Step 2 and increase $\epsilon_i$.

Prior to the further discussion, we introduce the two lemmas that are fundamental for establishing subsequent results.

*Lemma 5: Consider a system with the state-space realization*

$$\dot{x} = (A + \mu LC)x + Lu$$
$$y = Cx, \quad (40)$$

*and corresponding transfer function*

$$G(s) = C(sI - A - \mu LC)^{-1}L, \quad (41)$$

*where A is an anti-Hurwitz stable matrix (i.e. it has at least one eigenvalue in $\bar{\mathbb{C}}^+$), and the pair $(C, A)$ is detectable. Then, for every $\chi > \mu^{-1}$ there exists a gain matrix L such that $\|G\|_\infty < \chi$. Moreover, the gain matrix can be obtained as $L = -\mu^{-1}PC^T$, where $P > 0$ is a solution of the following ARE*

$$PA^T + AP + \epsilon I + \left(\frac{1}{\mu^2\chi^2} - 1\right)PC^T CP = 0, \quad (42)$$

*with $\epsilon$ being a positive constant.*

*Proof:* According to the bounded real lemma (BRL) [46], $A + \mu LC$ is Hurwitz stable and $\|G\|_\infty < \chi$, if there exists $P > 0$ satisfying

$$P(A + \mu LC)^T + (A + \mu LC)P + \frac{1}{\chi^2}LL^T + PC^T CP < 0. \quad (43)$$

Let $L = -\mu^{-1}PC^T$. Then, the inequality (43) becomes

$$PA^T + AP + \left(\frac{1}{\mu^2\chi^2} - 1\right)PC^T CP < 0. \quad (44)$$

Since $A$ has eigenvalues with nonnegative real parts, there does not exist $P > 0$ that satisfies the inequality (44) whenever $\mu^{-2}\chi^{-2} - 1 \geq 0$, thus $\chi > \mu^{-1}$ must hold.

The inequality (44) is satisfied for every $P > 0$ that is a solution of the ARE (42). The existence of such $P$ is ensured by detectability of $(C, A)$ [45]. □

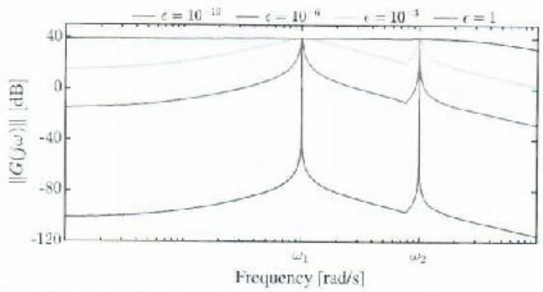



**FIGURE 1. Maximal singular value with respect to the frequency ($||G(j\omega)||$) of two-input, two-output system for $\mu^{-1} = 100$, $\chi = \mu^{-1} + 0.01$ and different values of $\epsilon$. The system is of the seventh order, with two pairs of poles on the imaginary axis, at $\pm j\omega_1$ and $\pm j\omega_2$.**

It should be noted that $-\mu^{-1}PC^T$ is not a unique substitution for the gain $L$ in terms of $P$. However, this choice of $L$ can lead to the minimum possible norm of $G(s)$. In the following, we establish the value of that minimum norm.

*Lemma 6: Consider the system given by (40), (41), where $A$ is an anti-Hurwitz stable matrix. Then, $||G||_\infty \geq \mu^{-1}$.*

*Proof:* Under the the feedback controller $u = -\mu y$, the closed loop matrix of (41) is equal to $A$, which is anti-Hurwitz stable by assumption. Therefore, according to the small-gain theorem, $||G||_\infty \geq \mu^{-1}$ must hold. □

*Remark 7: In the case when $A$ is an anti-Hurwitz stable matrix, Lemma 5 and Lemma 6 imply that there always exists a gain $L$ such that $\|G\|_\infty \in [\mu^{-1}, \chi)$, where $\chi$ can be chosen to be arbitrarily close to $\mu^{-1}$. It should be noted, that when $A$ is Hurwitz, there is no restriction on the lower bound of $\|G\|_\infty$ since the inequality can be satisfied for any $\chi > 0$.*

*Remark 8: Suppose that $\chi$ is chosen as $\chi = \mu^{-1} + \Delta$, where $\Delta > 0$ is a small constant. Then, for any $\epsilon > 0$, we have $\|G\|_\infty \approx \mu^{-1}$. Furthermore, it is well-known that the solution $P(\epsilon)$ of the ARE (42) is monotonically non-decreasing with respect to $\epsilon$. In the special case, when state matrix has eigenvalues in the closed left half-plane, $\epsilon \to 0$ corresponds to $P(\epsilon) \to 0$ [47], [48], thus leading to $L \to 0$.*

In order to illustrate the scenario in Remark 8, Fig. 1 shows the largest singular value with respect to frequency, i.e. $||G(j\omega)||$, for a marginally stable MIMO system. The system is of the seventh order, with two pairs of poles at $\pm j\omega_1$ and $\pm j\omega_2$. As it can be seen, $\|G\|_\infty \to \mu^{-1}$ for $\epsilon = 10^{-10}$, where the largest singular values occur at the frequencies $\omega_1$ and $\omega_2$. On the rest of the frequency range, $||G(j\omega)||$ decreases as frequencies get further from $\omega_1$ and $\omega_2$, due to $L \to 0$. An increase of the value of $\epsilon$ causes a larger gain matrix $L$, which further increases $||G(j\omega)||$ on the rest of the frequency range. However, $||G||_\infty$ remains the same.

### A. SOLVABILITY ANALYSIS UNDER ROF PROTOCOL

In this subsection we provide some additional results regarding the solvability of the COR problem under the ROF protocol.

First, introduce the following matrices

$$\check{H}_i = \begin{bmatrix} A_i & -B_i\Gamma_i^{\omega} & -B_i\Gamma_i^{\nu} \\ 0 & P & 0 \\ 0 & 0 & S \end{bmatrix}, \; \check{L}_i = \begin{bmatrix} I - \Pi_i^{\omega} - \Pi_i^{\nu} \, L_i \\ L_i^{\omega} \\ L_i^{\nu} \end{bmatrix},$$
$$\check{G}_i = \begin{bmatrix} -C_i & 0 & 0 \end{bmatrix}.$$

Then, by taking into account (19), the transfer function (14) can be rewritten in terms of these matrices as

$$\begin{aligned} T_i(s) &= C_i(sI - A_i - B_iK_i^x)^{-1}\left[I \; -\Pi_i^{\omega} \; -\Pi_i^{\nu}\right]L_i \\ &\quad + \left(I + \mu C_i(sI - A_i - B_iK_i^x)^{-1}\left[I \; -\Pi_i^{\omega} \; -\Pi_i^{\nu}\right]L_i\right) \\ &\quad \times \check{G}_i(sI - \check{H}_i - \mu\check{L}_i\check{G}_i)^{-1}\check{L}_i. \end{aligned} \tag{45}$$

Define the new transfer function $\bar{T}_i(s) = \check{G}_i(sI - \check{H}_i - \mu\check{L}_i\check{G}_i)^{-1}\check{L}_i$, which can be shown to be identical to $\bar{T}_i(s) = G_i(sI - H_i - \mu L_iG_i)^{-1}L_i$. This leads to a more concise representation of (45) in terms of matrices in original coordinates

$$\begin{aligned} T_i(s) &= C_i(sI - A_i - B_iK_i^x)^{-1}\left[I \; -\Pi_i^{\omega} \; -\Pi_i^{\nu}\right]L_i \\ &\quad + \left(I + \mu C_i(sI - A_i - B_iK_i^x)^{-1}\left[I \; -\Pi_i^{\omega} \; -\Pi_i^{\nu}\right]L_i\right) \\ &\quad \times \bar{T}_i(s). \end{aligned} \tag{46}$$

*Assumption 6:* The eigenvalues of $A_i$, $i = 1, \ldots, N$, belong to the closed left half-plane of the complex plane, while the eigenvalues of the matrices $P$ and $S$ lie on the imaginary axis.

*Theorem 3: Suppose that $\mu$ is chosen such that the condition (20) holds. Then, under Assumptions 1-6, the COR problem is always solvable by ROF protocol (7). Moreover, for a sufficiently small $\epsilon_i$, the values of $K_i$ and $L_i$, $i = 1, \ldots, N$, can be obtained by Algorithm 2.*

*Proof:* The ARE (25) in *Algorithm 2* corresponds to the ARE (42) when $\delta_i = 1 - \mu^{-2}\chi^{-2}$. The Assumption 6 implies that $H_i$ has eigenvalues in the closed left half-plane, thus by Remark 8, $L_i^{\nu} \to 0$ as $\epsilon_i \to 0$. Therefore, for any gain $K_i^x$ that stabilizes $A_i + B_iK_i^x$ we can write $T_i(s) \to \bar{T}_i(s)$. Choosing a sufficiently small $\delta_i$ in *Algorithm 2* leads to $\|\bar{T}_i\|_\infty \to \mu^{-1}$, thus implying $||T_i||_\infty \to \mu^{-1}$. The rest of the proof follows from *Corollary 1*. □

*Remark 9: The Theorem 3 guarantees the existence of a solution for agents with poles in the closed left half-plane for $\epsilon_i \to 0$. On the other side, the system response becomes faster as $\epsilon_i$ increases because $X_i(\epsilon_i)$, and hence the control input, also increase with $\epsilon_i$. Even though for a larger $\epsilon_i$ we still have $||\bar{T}_i||_\infty \to \mu^{-1}$, the other terms in $T_i(s)$ become non-negligible, leading to a larger $||T_i||_\infty$. This makes it less likely to find $K_i^x$ that solves the $\mathcal{H}_\infty$ SOF problem. Furthermore, it should be noted that if either an agent or an exosystem has poles in the open right half-plane, the existence of a solution cannot be guaranteed in advance, as it is not possible to achieve $L_i \to 0$ for any $\epsilon_i$. Namely, as the unstable poles of the exosystems or agents move further right in the complex plane, the required stabilizing gain $L_i$ increases. Consequently, minimizing $\|T_i\|_\infty$, which depends on both $L_i$ and the follower state model, becomes more difficult. It is* *important to note that, in general, the $\mathcal{H}_\infty$ norm of linear systems cannot be arbitrarily reduced by using state or output feedback [49].*

*Remark 10: Although seemingly restrictive, the Assumption 6 covers a wide variety of important and common agents' dynamics in multi-agent systems. An example are agents with first and second order integrator dynamics [10], [11]. The assumption that the the poles of exosystems lie on imaginary axis is a common assumption in many existing results such as [7], [16], [18], [21], [31], [32], [39], and [37]. Under the assumption, the exosystems associated with matrices S and P can generate a diverse range of reference and disturbance signals that are interesting in practice. This includes step signals, polynomial signals, sinusoidal signals of various frequencies, and their linear combinations [31]. Possible practical applications include cooperative tracking and formation control of mobile vehicles [2], [39], control of unmanned aerial vehicles [33], control of robotic manipulators [20], and so on.*

*Remark 11: Solvability of the COR problem for certain classes of agents' dynamics has also been discussed in related works. For example, in [27] it is assumed that the agents are minimum-phase and with an identical relative degrees. In [37], an $\mathcal{H}_\infty$ based design method is proposed, but it does not guarantee that the stabilizing controller gains can be found, even when the agents' dynamics is stable. Similarly, [28] considers the output synchronization of the non-introspective agents that are minimum-phase and SISO.*

### B. SOLVABILITY ANALYSIS UNDER OF PROTOCOL

In this subsection, we analyze solvability of the COR problem under the OF protocol.

In terms of the matrices introduced in (33), the transfer function (30) can be rewritten as

$$T_i(s) = -C_i\big(sI - (A_i + B_iK_i^x)\big)^{-1}\Pi_i^\nu L_i^\nu + \Big(I - \mu C_i\big(sI - (A_i + B_iK_i^x)\big)^{-1}\Pi_i^\nu L_i^\nu\Big)\,\bar{T}_i(s), \tag{47}$$

where $\bar{T}_i(s) = F\big(sI - (S + \mu L_i^\nu F)\big)^{-1}L_i^\nu$, which corresponds to the form in *Lemma 5*.

*Assumption 7:* The eigenvalues of the matrix $S$ lie on the imaginary axis.

*Theorem 4: Suppose that $\mu$ is chosen such that the condition (20) holds. Then, under Assumptions 1-5 and 7, the COR problem is always solvable by OF protocol (26). Moreover, for a sufficiently small $\epsilon_i$, the values of $K_i$ and $L_i$, $i = 1, \ldots, N$, can be obtained by Algorithm 3.*

*Proof:* The ARE (39) in *Algorithm 3* corresponds to the ARE (42) when $\delta_i = 1 - \mu^{-2}\chi^{-2}$. Under the Assumption 7, $S$ is marginally stable, thus by Remark 8, $L_i \to 0$ as $\epsilon_i \to 0$. Therefore, for any gain $K_i^x$ that stabilizes $A_i + B_iK_i^x$ it follows that $T_i(s) \to \bar{T}_i(s)$. The choice of a sufficiently small $\delta_i$ in *Algorithm 3* leads to $\|\bar{T}_i\|_\infty \to \mu^{-1}$, implying $||T_i||_\infty \to \mu^{-1}$. The rest of the proof follows from *Corollary 2*. □

*Remark 12: Along with Assumption 7, some existing results require additional assumptions in order to guarantee the solvability of the COR problem. For example, the observer-based low-gain method [31] guarantees the existence of a solution only when $E_i = 0$, while the approach [36] requires the agents to be right-invertible. Furthermore, contrary to some existing works, the proposed method does not impose any restrictions on the spectrum of the matrix P.*

Finally, for the sake of clarity and to avoid repetition, we note that conclusions can be drawn in this section analogously to those presented in Remark 9.

## VI. SIMULATION RESULTS

### A. EXAMPLE 1

Consider a network consisting of a leader and six followers with the following dynamics

$$\begin{cases} \dot{\nu} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\nu, \; y_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\nu, \\ \dot{\omega} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\omega, \end{cases}$$

$$\begin{cases} \dot{x}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}x_i + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}u_i + \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}\omega, \\ y_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x_i + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\omega, \; i = 1, 4; \end{cases}$$

$$\begin{cases} \dot{x}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}x_i + \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}u_i + \begin{bmatrix} -2 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix}\omega, \\ y_i = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}x_i + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\omega, \; i = 2, 5; \end{cases}$$

$$\begin{cases} \dot{x}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -36 & 0 \end{bmatrix}x_i + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}u_i + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\omega, \\ y_i = \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & -1 & 1 \end{bmatrix}x_i + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\omega, \; i = 3, 6. \end{cases}$$

The solutions of the regulator equations (6) for agents $i = 1, 4$, $i = 2, 5$, and $i = 3, 6$ are respectively:

$$\Gamma_i^\omega = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}, \; \Gamma_i^\nu = \begin{bmatrix} 0 & -2 \\ -1 & 0 \end{bmatrix}, \; \Pi_i^\omega = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix},$$
$$\Pi_i^\nu = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\Gamma_i^\omega = \begin{bmatrix} 0 & 3.5 \\ -5 & 3 \end{bmatrix}, \; \Gamma_i^\nu = \begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix}, \; \Pi_i^\omega = \begin{bmatrix} 1 & 1.5 \\ -1 & -2.5 \\ 1 & 2.5 \end{bmatrix},$$
$$\Pi_i^\nu = \begin{bmatrix} -1 & 1 \\ 2 & -1 \\ -2 & 2 \end{bmatrix},$$
$$\Gamma_i^\omega = \begin{bmatrix} 0.67 & 0.33 \\ 0 & -13.33 \end{bmatrix}, \; \Gamma_i^\nu = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \; \Pi_i^\omega = \begin{bmatrix} 0 & -0.83 \\ 0 & -0.33 \\ 0 & -0.33 \end{bmatrix},$$
$$\Pi_i^\nu = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The network topology among agents is described by

$$\bar{\mathcal{L}} = \begin{bmatrix} [r]1 & -0.10 & 0 & -0.1 & 0 & -0.8 \\ -0.12 & 1 & -0.12 & 0 & -0.12 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -0.67 & -0.33 & 0 & 1 & 0 & 0 \\ 0 & -0.8 & 0 & 0 & 1 & -0.2 \\ 0 & -0.33 & 0 & 0 & -0.33 & 1 \end{bmatrix}.$$

By setting $\mu = 1$, for the network topology defined by $\bar{\mathcal{L}}$, we obtain $\gamma^* = \rho(\mu I - \bar{\mathcal{L}})^{-1} = 1.51$. Based on Theorems 1 and 2, in order to guarantee the stability of the MAS, the $\mathcal{H}_\infty$ norms $\|T_i(s)\|_\infty$, for $i = 1, \dots, N$, must be smaller than $\gamma^*$.

1) ROF PROTOCOL

The observer gains are calculated by solving ARE (25) in Algorithm 2 for $\epsilon_i = 10$ and $\delta_i = 0.1$. The resulting observer gains are:

$$L_i = \begin{bmatrix} 19.17 & 9.99 & -9.25 & 1.97 & -6.32 & 6.18 \\ -2.59 & 9.09 & -9.30 & 4.91 & -1.49 & -10.94 \end{bmatrix}^T, \; i = 1, 4,$$
$$L_i = \begin{bmatrix} 28.11 & 15.48 & -9.98 & -2.01 & 10.58 & -0.25 & 8.85 \\ -16.60 & 11.01 & 0.32 & 3.48 & -8.48 & 1.25 & -10.95 \end{bmatrix}^T, \; i = 2, 5,$$
$$L_i = \begin{bmatrix} 5.07 & 1.46 & 0.56 & -8.24 & 10.98 & -6.69 & 2.04 \\ -5.81 & -6.55 & 45.76 & -2.40 & 2.41 & -1.31 & -12.22 \end{bmatrix}^T, \; i = 3, 6.$$

Then, the gain $K_i^x$ for each agent is determined using Algorithm 1, which is implemented in YALMIP, a MATLAB optimization toolbox. For $\gamma_i = 1.2$, the following gains are obtained:

$$K_i^x = \begin{bmatrix} -4.95 & -0.60 \\ -0.60 & 2.26 \end{bmatrix}, \; i = 1, 4,$$
$$K_i^x = \begin{bmatrix} -2.39 & -3.38 & -0.59 \\ 2.55 & 3.50 & -0.15 \end{bmatrix}, \; i = 2, 5,$$
$$K_i^x = \begin{bmatrix} -2.49 & -3.10 & 0.00 \\ 2.39 & 39.35 & -4.71 \end{bmatrix}, \; i = 3, 6.$$

In the final step, the gains $K_i^\omega$ and $K_i^\nu$ are calculated as $K_i^\omega = \Gamma_i^\omega - K_i^x \Pi_i^\omega$ and $K_i^\nu = \Gamma_i^\nu - K_i^x \Pi_i^\nu$, respectively. The resulting $\mathcal{H}_\infty$ norms are: $\|T_i\|_\infty = 1.14$ for $i = 1, 3$, $\|T_i\|_\infty = 1.12$ for $i = 2, 5$ and $\|T_i\|_\infty = 1.15$ for $i = 3, 6$. Since these norms are smaller than $\gamma^*$, we can conclude that the COR problem is solved.

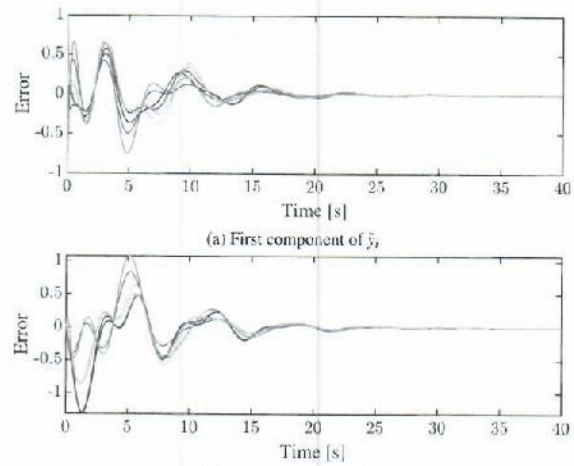
In Fig. 2, the output regulation errors of the agents under the designed controller are shown. It can be seen that the ROF protocol ensures tracking of the reference signal, even in the presence of disturbance. Furthermore, in Fig. 3 the output trajectories are depicted, demonstrating that all outputs synchronize with the reference trajectory.

2) OF PROTOCOL

In the case of the OF protocol, by solving the ARE (39) in Algorithm 3 for $\epsilon_i = 10$ and $\delta_i = 0.1$, we obtain the following distributed observer gains:

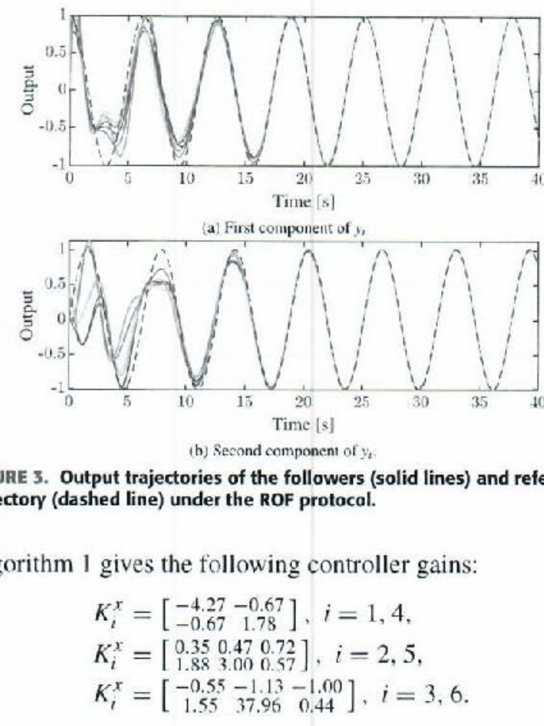
$$L_i^\nu = \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix}, i = 1, \dots, N.$$

Similarly, the gains of the local observer are obtained by solving the ARE (38) for $\kappa_i = 1$, but their specific values are not presented for the sake of brevity. For $\gamma_i = 1.2$, the




(b) Second component of $\tilde{y}_i$.

**FIGURE 2. Output regulation errors of the followers under the ROF protocol.**




(b) Second component of $y_i$.

**FIGURE 3. Output trajectories of the followers (solid lines) and reference trajectory (dashed line) under the ROF protocol.**

Algorithm 1 gives the following controller gains:

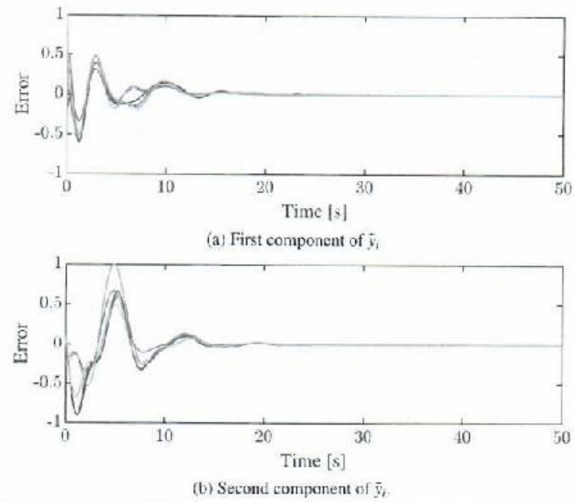
$$K_i^x = \begin{bmatrix} -4.27 & -0.67 \\ -0.67 & 1.78 \end{bmatrix}, \; i = 1, 4,$$
$$K_i^x = \begin{bmatrix} 0.35 & 0.47 & 0.72 \\ 1.88 & 3.00 & 0.57 \end{bmatrix}, \; i = 2, 5,$$
$$K_i^x = \begin{bmatrix} -0.55 & -1.13 & -1.00 \\ 1.55 & 37.96 & 0.44 \end{bmatrix}, \; i = 3, 6.$$

The remaining gains are determined as $K_i^\omega = \Gamma_i^\omega - K_i^x \Pi_i^\omega$ and $K_i^\nu = \Gamma_i^\nu - K_i^x \Pi_i^\nu$. As a result, we have: $\|T_i\|_\infty = 1.19$ for $i = 1, 3$, $\|T_i\|_\infty = 1.18$, for $i = 2, 5$ and $\|T_i\|_\infty = 1.003$ for $i = 3, 6$. Since the resulting norms are smaller than $\gamma^*$, we conclude that the COR problem is solved.

Figs. 4 and 5 show the output regulation errors and output trajectories under the designed OF protocol. It can be seen that the errors asymptotically converge to zero, while agent outputs synchronize with the reference trajectory, indicating successful output regulation.

### B. EXAMPLE 2

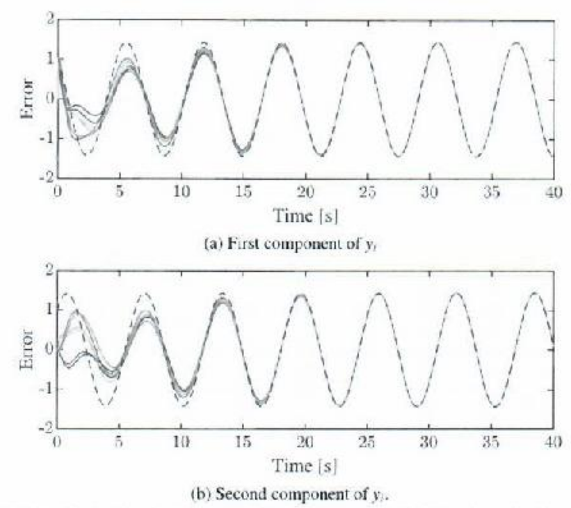
In this example, we consider the MAS from Example 1 and investigate the influence of various design parameters on closed-loop performance for both the ROF and OF protocols.




(a) First component of $\tilde{y}_i$

(b) Second component of $\tilde{y}_i$.

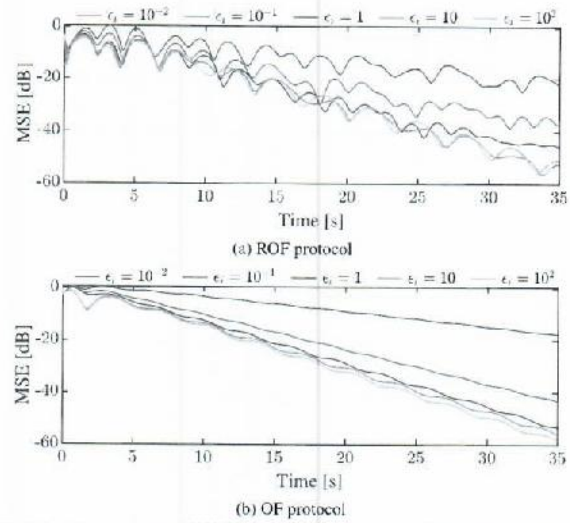
**FIGURE 4. Output regulation errors of the followers under the OF protocol.**




(a) First component of $y_i$

(b) Second component of $y_i$.

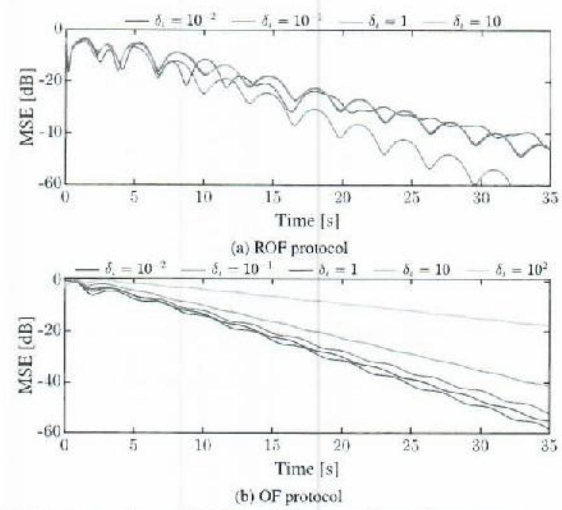
**FIGURE 5. Output trajectories of the followers (solid lines) and reference trajectory (dashed line) under the OF protocol.**




(a) ROF protocol

(b) OF protocol

**FIGURE 6. Comparison of MSE for different values of parameter $\epsilon_i$.**




(a) ROF protocol

(b) OF protocol

**FIGURE 7. Comparison of MSE for different values of parameter $\delta_i$.**

In the first scenario, the parameter $\epsilon_i$ is varied, while other parameters are the same as in the previous example. Fig. 6 shows the mean squared output regulation error for all agents, which is calculated as $\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}\tilde{y}_i^T\tilde{y}_i$. The figure indicates that increasing $\epsilon_i$ enhances the performance of the MAS. However, for $\epsilon_i = 10$ and $\epsilon_i = 100$, the difference in performance is not too significant, meaning that the excessive values for $\epsilon_i$ will not improve the performance substantially. It is important to note that by increasing $\epsilon_i$, the gains $L_i$ (ROF) and $L_i^y$ (OF) also increase, and for some high values (e.g., $10^4$ in this example), the SOF algorithm will not be able to provide a solution.

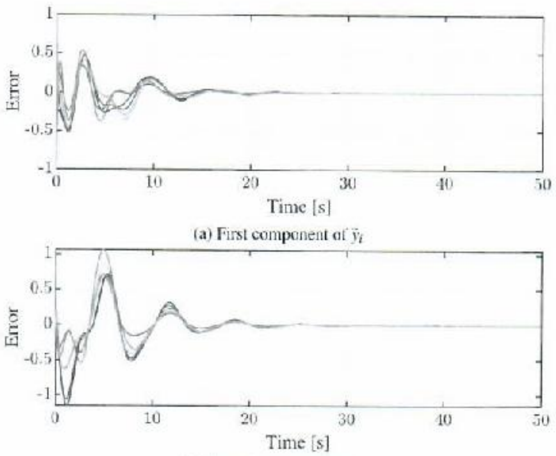
In the second scenario, the parameter $\delta_i$ is varied, while the other parameters are kept the same as in the previous example. In the case of the ROF protocol, it was not possible to find $K_i^x$ using the SOF algorithm for $\delta_i = 100$. The resulting MSE curves are depicted in Fig. 7. Notably, for the ROF protocol, the output regulation errors decay at the fastest rate when $\delta_i = 0.1$, while in the case of the OF protocols, the best performances are obtained for $\delta_i = 0.01$. Clearly, $\delta_i$ can be used to adjust performance, but excessive values can lead to performance deterioration.

### C. EXAMPLE 3

In this example, we consider exosystems that have poles with strictly positive real parts to demonstrate that even when the existence of a solution cannot be guaranteed in advance, the proposed method still may solve the COR problem, as discussed in Remark 9. The exosystem matrices are given by: $S = \begin{bmatrix} 0.5 & 1 \\ 0 & -1 \end{bmatrix}$ and $P = \begin{bmatrix} 0.5 & 2 \\ 0 & -2 \end{bmatrix}$, while the followers' model and parameters of Algorithms 1-3 remain unchanged.

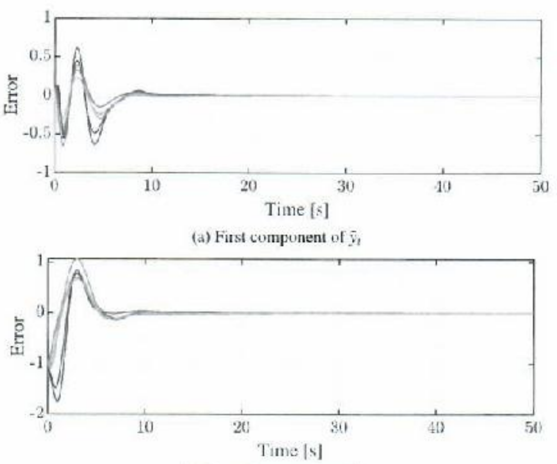
In the case of the ROF protocol, the following gains and $\mathcal{H}_\infty$ norms are obtained:

$$K_i^x = \begin{bmatrix} -0.35 & -0.38 \\ -0.38 & 5.76 \end{bmatrix}, \ \|T_i\|_\infty = 1.19, \ i = 1, 4,$$
$$K_i^x = \begin{bmatrix} 1.25 & -1.18 & -1.66 \\ 22.14 & 15.22 & -5.70 \end{bmatrix}, \ \|T_i\|_\infty = 1.18, \ i = 2, 5,$$
$$K_i^x = \begin{bmatrix} 1.25 & -1.18 & -1.66 \\ 22.14 & 15.22 & -5.70 \end{bmatrix}, \ \|T_i\|_\infty = 1.20, \ i = 3, 6,$$




(b) Second component of $\tilde{y}_i$.

**FIGURE 8. Output regulation error under the ROF protocol in the case when the exosystem matrices are unstable.**




(b) Second component of $\tilde{y}_i$.

**FIGURE 9. Output regulation error under the OF protocol in the case when the exosystem matrices are unstable.**

while in the case of the OF protocol we have:

$$K_i^x = \begin{bmatrix} -6.51 & -1.17 \\ -1.17 & 6.44 \end{bmatrix}, \ \|T_i\|_\infty = 1.12, \ i = 1, 4,$$
$$K_i^x = \begin{bmatrix} 1.55 & -0.47 & -1.54 \\ 7.49 & 4.15 & -4.00 \end{bmatrix}, \ \|T_i\|_\infty = 1.17, \ i = 2, 5,$$
$$K_i^x = \begin{bmatrix} -11.04 & -7.98 & 0.54 \\ 9.84 & 47.20 & -5.98 \end{bmatrix}, \ \|T_i\|_\infty = 1.10, \ i = 3, 6.$$

Figures 8 and 9 show the output regulation errors of the followers. It can be seen that, despite the instability of the exosystem, the errors asymptotically converge to zero. The output trajectories are not shown as they exhibit exponential growth.

## VII. CONCLUSION

In this paper, we have proposed a novel observer-based approach for solving the COR problem for heterogeneous MASs. Two COR protocols have been presented, for networks of non-introspective and networks of introspective agents, respectively. The proposed protocols do not require the exchange of the controller states and thus reduce the communication burden. Furthermore, algorithms based on $\mathcal{H}_\infty$ SOF theory and ARE methods are provided for determining the controller gains. It has been proven that for a large class of reference signals the solvability of the COR problem can be guaranteed in advance for: i) introspective agents with arbitrary dynamics, ii) non-introspective agents with stable dynamics.

The future work will be focused in two directions. The first direction is extension of the proposed method to dynamically switching networks as well achieving robust performance in a presence of communication delays in the case of introspective agents. The second direction is investigation of the possibility of applying the proposed method to the leaderless output synchronization problem and output synchronization problem over signed graphs.

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**А. Биографски подаци**

Жељко Ђуровић је рођен 28.03.1964. године у Пироту. На Електротехничком факултету у Београду је дипломирао 1988. године на Одсеку за аутоматику. Исте године је уписао магистарске студије на Одсеку за мерење и управљање и магистрирао је 1989. године. Докторску дисертацију је одбранио 1994. године на тему *Адаптивна и робусна естимација стања стохастичких система,* под руководством проф. Бранка Ковачевића а, такође, на Електротехничком факултету у Београду.

Непосредно након дипломирања провео је неко време као развојни инжењер у Лабораторији за радаре при Истраживачком развојном институту за телекомуникације (ИРИТЕЛ) у Београду, бавећи се проблемима праћења више покретних циљева помоћу радарских система. На Електротехничком факултету у Београду се запослио октобра 1988. године у звању асистент-приправник. Каријеру асистента, а затим и доцента је на истом Факултету прекинуо 2000. године, када је отишао на Државни технички универзитет у Лисабону, на позицију пост-доктoранта у Институту за системе и роботику. На овом Институту се бавио применом техника статистичког препознавања облика у индустрији. Након тога, проводи две године у компанији *Visteon gmbh,* у Келну, Савезна Република Немачка, на позицији специјалисте за регулацију. Током свог боравка у овој компанији се бавио развојем регулације за HVAC системе у аутомоблилској индустрији који користе угљен-диоксид као термодинамички медијум.

Крајем 2002. године се враћа као доцент на Електротехнички факултет у Београду, где 2004. године бива биран за ванредног професора а затим 2010. године и за редовног професора. У периоду од 2002. до 2004. обавља функцију продекана за финансије и председника Савета факултета у периоду од 2012. до 2018. године. У више наврата је био шеф Катедре за сигнале и системе, као и шеф Одсека. У два мандата је обављао функцију председника Комисије за студије првог степена. Члан је Матичног одбора за електронику, телекомуникације и рачунарску технику при ресорном Министарству.

На Катедри за сигнале и системе на Електротехничком факултету у Београду држи наставу из групе предмета везаних за управљање системима и обраду сигнала. Руководио је великим бројем дипломских и мастер радова, и извео је петнаест доктораната. Основне области његовог истраживања су теорија стохастичких система, теорија естимације и примене, а конкретно то су проблеми у оквиру управљања индустријским процесима, детекција и изолација отказа у техничким системима, развој и унапређење система за праћење покретнихних циљева и специфичне технике у обради сигнала. Према Scopus бази, радови Жељка Ђуровића су цитирани 909 пута и има *h* индекс 14.

**Б. Библиографија научних и стручних радова у периоду 2015-2024.**

*Радови у међународним часописима (М20)*


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У Београду,
Дана 19.03.2024.

Ж. Ђуровић

Др Жељко Ђуровић, ред. професор

ЕЛЕКТРОТЕХНИЧКИ ФАКУЛ...
БЕОГРАД
ПРИМЉЕНО 27 JAN 2010
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**УНИВЕРЗИТЕТ У БЕОГРАДУ**

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Београд, 19.01.2010. године
06 Број:150-58/V-2.3
JJ

На основу чл. 65. ст. 2. Закона о високом образовању ("Службени гласник РС", број 76/05, 97/08 и 100/07-аутентично тумачење), чл. 41. ст. 1. тач. 23 и чл. 42. ст. 4. Статута Универзитета у Београду ("Гласник Универзитета у Београду", број 131/06, 140/08, 143/08 и 150/09), чл. 25. ст. 1. и ст. 2. тач. 1. Правилника о начину и поступку стицања звања и заснивања радног односа наставника Универзитета у Београду ("Гласник Универзитета у Београду", број 142/08) и Критеријума за стицање звања наставника на Универзитету у Београду ("Гласник Универзитета у Београду", број 140/08), а на предлог Изборног већа Електротехничког факултета, број: 1484/4 од 05.10.2009. године, и мишљења Већа научних области техничких наука, број: 011-612-25/186/09 са седнице од 20.11.2009. године, Сенат Универзитета, на седници одржаној 19. јануара 2010. године, донео је

**ОДЛУКУ**

**БИРА СЕ** проф. др ***Жељко Ђуровић*** у звање редовног професора на Универзитету у Београду – Електротехнички факултет, за ужу научну област Аутоматика.

*Образложење*

Електротехнички факултет је дана 08. јула 2009. године, у листу «Послови», објавио конкурс за избор у звање редовног професора, за ужу научну област Аутоматика, због потреба факултета.

Извештај Комисије за припрему извештаја о пријављеним кандидатима стављен је на увид јавности дана 03. септембра 2009. године преко библиотеке Факултета.

На основу предлога Комисије за припрему извештаја о пријављеним кандидатима, Изборно веће Факултета, на седници одржаној дана 22. септембра 2009.године, донело је одлуку о утврђивању предлога да се кандидат др Жељко Ђуровић изабере у звање редовног професора.

Електротехнички факултет је дана 06. октобра 2009.године доставио Универзитету комплетан захтев за избор у звање на прописаним обрасцима.

Универзитет је комплетну документацију коју је доставио факултет ставио на web страницу Универзитета дана 13. новембра 2009. године.

Веће научних области техничких наука, на седници одржаној дана 20. новембра 2009. године дало је мишљење да се проф. др Жељко Ђуровић може изабрати у звање редовног професора.

Сенат Универзитета, на седници одржаној дана 19. јануара 2010. године разматрао је захтев Електротехничког факултета и утврдио да кандидат испуњава услове прописане чл. 64. и 65. Закона о високом образовању и чланом 120. Статута Универзитета у Београду, па је донета одлука као у изреци.

ПРЕДСЕДНИК СЕНАТА
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- **Kratka biografija**

Rođen sam 7. aprila 1968. god. u Žabljaku, gdje sam završio osnovnu školu i prva dva razreda srednjeg usmjerenog obrazovanja. Srednju školu sam završio u gimnaziji "Slobodan Škerović" u Podgorici.Na Elektrotehničkom fakultetu u Podgorici sam diplomirao marta 1992. godine sa prosječnom ocjenom 9,87, a diplomski rad "YAMABICO - upravljanje mobilnim robotom" sam odbranio sa ocjenom 10. Dobitnik sam studentske nagrade "19. decembar" i Plakete Univerziteta kao najbolji student Univerziteta 1991. god. Postdiplomske studije sam upisao na istom fakultetu 1992. godine, na Odsjeku robotike i vještačke inteligencije. Ispite na postdiplomskim studijama sam položio sa prosječnom ocjenom 10, a magistarski rad pod nazivom "Modifikovani adaptivni LMS algoritmi" sam odbranio 1996. godine. Doktorsku disertaciju, pod nazivom "Novi pristup LMS adaptivnom algoritmu sa promjenljivim korakom", odbranio sam 20. 12. 2002. godine na Elektrotehničkom fakultetu u Podgorici.

U zvanje docenta sam izabran 09.07.2003. godine, u zvanje vanrednog profesora 02.10.2008. godine, a zvanje redovnog profesora 19.12.2013. godine na Univerzitetu Crne Gore.

Autor sam ili koautor dvije monografije, više udžbenika za osnovnu školu iz oblasti informatike i više autorizovanih skripti za potrebe nastave na predmetima na kojima je angažovan. Do sada sam objavio preko 150 naučnih i stručnih radovan u časopisima i na konferencijama. Pod mojim mentorstvom su uspješno završena: 4 doktorska, 15 magistarskih i preko 150 diplomskih i specijalističkih radova. Recenzirao sam više naučnih radova u istaknutim svjetskim časopisima iz oblasti adaptivnih algoritama i računarskih sistema.

Koordinirao sam i učestvovao u više značajnih evropskih projekata kao predstavnik Univerziteta Crne Gore, a koje finansira Evropska unija u okviru FP6, FP7, TEMPUS, IPA, EUREKA i H2020 programa (SEEREN2, SEE-GRID2, SEE-GRID-SCI, SEERA-EI, GEANT3, NQF&QHE, GEANT3+, HPSEE, EGI-Inspire, DL@WEB, RINGINDEA, FORSEE, CONGRAD, GN4, VI-SEEM, FASTER i NI4OS). Angažovan sam od strane više kompanija i institucija u Crnoj Gori i van nje kao stručni ICT konsultant, projektant ili ekspert, te sam projektovao, nadzirao i realizovao više značajnih stručnih projekata. Od strane sudova u Crnoj Gori sam angažovan kao sudski vještak za oblast ICT-a. Bio sam preko 12 godina direktor Centra informacionog sistema UCG-a.

Predsjednik sam organizacionog i programskog odbora domaćeg naučno-stručnog skupa »INFORMACIONE TEHNOLOGIJE« koja se već 25 godine organizuje i editor sam zadnjih 11 zbornika ove konferencije. Takodje, član sam programskih odbora tri međunarodne konferencije: "Balkan Conference in Informatics, "RoEduNet Conference: Networking in Education and Research" i „ETIMA 2021" kao i član Predsjedništva društva ETRAN. Član sam međunarodne asocijacije elektro inženjera – IEEE, inženjerske komore Crne Gore, Internet zajednice ISOC i Ubuntu zajednice Crne Gore. Menadžer sam nacionalnog .me domena (ccTLD manager).Pokretač sam MREN-a (Montenegrin Research and Education Network) i MIXP-a (Montenegro Internet eXchange Point) i član njihovih odbora.

Govorim engleski jezik, a služim se i ruskim jezikom.

- **Radovi iz oblasti objavljeni u časopisima sa SCI liste u zadnjih 10 godina:**

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Na osnovu člana 75 stav 2 Zakona o visokom obrazovanju (Sl.list RCG, br. 60/03 i Sl.list CG, br. 45/10 i 47/11) i člana 18 stav 1 tačka 3 Statuta Univerziteta Crne Gore, Senat Univerziteta Crne Gore, na sjednici održanoj 19.12.2013. godine, donio je

**ODLUKU**
**O IZBORU U ZVANJE**

**Dr BOŽO KRSTAJIĆ** bira se u akademsko zvanje **redovni profesor** Univerziteta Crne Gore za predmete: Operativni sistemi, osnovne studije–ETR, Adaptivni sistemi upravljanja-specijalističke studije EA, Modelovanje i simulacija dinamičkih sistema-specijalističke studije EA, **na Elektrotehničkom fakultetu** i Automatsko upravljanje, na Mašinskom fakultetu.

**REKTOR**
Prof.dr Predrag Miranović

УНИВЕРЗИТЕТ ЦРНЕ ГОРЕ
ЕЛЕКТРОТЕХНИЧКИ ФАКУЛТЕТ
Број 02/1-2132
Подгорица, 25.12.2013 год

**Prof. dr Žarko Zečević**
**Elektrotehnički fakultet**
**Univerzitet Crne Gore**

## BIOGRAFIJA

Žarko Zečević je rođen 01.01.1989. godine u Nikšiću, gdje je završio osnovnu školu i gimnaziju. Za postignute rezultate u toku školovanja je nagrađen diplomom „Luča". Elektrotehnički fakultet u Podgorici je upisao 2007. godine. Osnovne studije na odsjeku za Elektroniku, Telekomunikacije i Računare je završio u junu 2010. godine sa prosječnom ocjenom 9,56. Specijalističke studije je završio u junu 2011. godine sa prosječnom ocjenom 10,00. Tokom studija je dva puta nagrađivan od strane fakulteta nagradom za najbolje studente u prethodnoj školskoj godini, i jednom od strane Univerziteta Crne Gore nagradom za najboljeg studenta Elektrotehničkog fakulteta 2007. godine.

Magistarske studije na Elektrotehničkom fakultetu u Podgorici, smjer Računari, je upisao 2011. godine, a završio ih 2012. godine sa prosječnom ocjenom 10,00. Magistarski rad pod nazivom "Poboljšanje performansi FxLMS-a u uslovima bijelog šuma" je odbranio 29.10.2012. Na istom fakultetu je upisao doktorske studije 2012. godine, iz oblasti istraživanja Automatika. Doktorsku tezu pod nazivom "Predlog novog adaptivnog algoritma za poboljšanje performansi ANC sistema" je odbranio 05.10.2015. godine.

Kao saradnik u nastavi na Elektrotehničkom fakultetu u Podgorici je anagažovan od decembra 2010. godine, u zvanje docenta je izabran u decembru 2016. godine, dok je u zvanje vanrednog profesora izabran u decembru 2021. godine.

Recenzent je u brojnim međunarodnim naučnim časopisima, među kojima su časopisi: IEEE Transactions on Systems, Man, and Cybernetics: Systems, IEEE Access, International Journal of Adaptive Control and Signal Processing, itd.

Objavio je preko 60 naučnih radova u časopisima i na konferencijama, od čega 14 radova u renomiranim svjetskim časopisima sa SCI/SCIE liste. U toku dosadašnjeg rada je učestvovao u realizaciji više međunarodnih naučnoistraživačkih projekata koje finansira Evropska unija u okviru IPA, EUREKA, Horizon 2020 i Horizon Europe programa (MONUSEN, NI4OS, VI-SEEM, FASTER, MARBLE). Pod njegovim mentorstvom odbranjeno je više magistarskih i specijalističkih radova. Koautor je dva nacionalna patenta.

Član je organizacionog odbora domaćeg naučno-stručnog skupa „Informacione Tehnologije". Takođe je član IEEE – međunarodnog udruženja inženjera elektrotehnike.

## SPISAK RADOVA PUBLIKOVANIH U ČASOPISIMA SA SCI/SCIE LISTE

[1] L. Martinović, **Ž. Zečević**, B. Krstajić, "Output containment control in heterogeneous multi-agent systems without exchange of controller states", European Journal of Control, vol. 75, 2024, 100889, ISSN 0947-3580, https://doi.org/10.1016/j.ejcon.2023.100889.

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[11] T. Popović, N. Latinović, A. Pešić, **Ž. Zečević**, B. Krstajić, S. Djukanović, „Architecting an IoT-enabled platform for precision agriculture and ecological monitoring: A case study, Computers and Electronics in Agriculture", Volume 140, 2017, pp. 255-265, ISSN 0168-1699, https://doi.org/10.1016/j.compag.2017.06.008.

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**O D L U K U**
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**Dr Žarko Zečević** bira se u akademsko zvanje vanredni profesor Univerziteta Crne Gore za oblast **Automatika** na Elektrotehničkom fakultetu Univerziteta Crne Gore, na period od pet godina.

**SENAT UNIVERZITETA CRNE GORE**
**PREDSJEDNIK**

**Prof. dr Vladimir Božović, rektor**